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An economic and statistical analysis of quality in high school education: the case of Iowa

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AN ECONOMIC AND STATISTICAL ANALYSIS OF QUALITY IN HIGH SCHOOL EDUCATION: THE CASE OF IOWA

by

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CHAPTER ONE. INTRODUCTION

Problems of educational policy have recently received a great deal of attention by economists, and there is a clear trend toward the intensification of research in the economics of education.[1]

Although much of the published research in this area concerns the educational sector in its entirety, e.g., for the U.S. as a whole, attempts have already been made to study the efficiency of the educational sector in more limited geographical areas. This study is an attempt to shed some additional light, in the context of the latter framework, on two major problems of educational research and policy:

1. What factors might determine the quality of high school education and how can that "quality" be measured?
2. Are there economies of scale in high school operations?

It is to be emphasized at the outset that our main purpose is the building and testing of models which could be used by individuals or organizations concerned with educational policy. The specific results which are reported in

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[1]An interesting insight may be obtained by noting that out of 19 Ph.D. candidates who will be available for positions in 1968 in the Economics Department of Iowa State University, 3 will have written their dissertations in the area of the economics of education.

Chapters 3, 4, and 5 can serve merely as an indication of how such analyses might be made. The data used in those chapters are for the years 1962-63, and the results cannot be taken as representative of the current [1968] situation in Iowa.

In what follows, we shall treat the educational establishment (high schools, in our study) as an industry, with each school district as a (multi- or single-plant) firm. Each firm produces a final product, and in the process it uses various types of fixed and variable inputs. Further, the inputs, X_i $(i = 1, \ldots, n)$, are transformed into the final product, Y, by a production function, f, where,

$$(1\text{-}1) \qquad Y = f(X_1, \ldots, X_n)$$

To simplify the analysis it might be assumed that a production function of <u>type</u> 1-1 holds for each of the firms in the industry, even though much variation may exist in the coefficients of each firm's function. (In other words, we may assume that for all firms f is homogeneous of degree 0 in the X_i, but not that f is <u>identical</u> to all firms.)

It must be emphatically noted that the final output (Y) of a high school is not analogous to that of a soap factory. For not only does the student gain skills and knowledge which can, perhaps, be measured to an extent by appropriate tests; he also gains cultural, civic, and perhaps moral values which cannot so easily be measured. Therefore, the composite scores on the Iowa Tests of Educational Development (referred hereafter as ITED) used as a proxy for the final product in Chapter 3 can at best measure only a part of the total product.

The Data

The empirical information used in the testing of models in this essay was, for the most part, compiled by the Iowa Department of Public Instruction. However, much of the information was thereafter processed and rearranged by Dr. Robert W. Thomas. In the original data set there were many more units of observation (i.e., high school districts) than have been used in the final set, since some of the variables under consideration were not reported for all districts. Nevertheless, we have retained the majority of the approved 4-year Iowa high schools (in existence in 1962-63) so that we may relate the results of the analysis to the Iowa high school "system."

The Iowa Tests of Educational Development

The test battery of the ITED is composed of nine different examinations, of the objective type, "designed to provide a comprehensive and dependable description of the general educational development of the high school pupil" (21, p. 6). The examinations deal with basic social concepts, the natural sciences, quantitative analysis, reading, expression, vocabulary, and other areas. While the tests are "intended for administration in an annual testing program to all pupils in grades 9 through 12," they may actually be administered only once every two years (e.g., in the 9th and 11th grades or in the 10th and 12th grades). An important feature of the ITED is that "the test results for the various years [are] directly comparable to one another," thus enabling us to make progress studies (21, p. 8).

The test results are given in a number of ways. First, each student receives a profile of the results on the nine tests, including a composite score which is a weighted average of the results on the first eight tests (thus excluding the test entitled Use of Sources of Information). Also, profiles and composite scores are given for each class, and, finally, similar results are reported for the school. In our study, we shall concentrate only upon the composite score for the school as a whole--for a given grade--since this is the only test result which was made available to us.

It is to be noted that the 12th grade score, for example, "must be regarded as a result of a lifetime of educational experiences, both in and out of school.... For example, a school's 10th grade performance on Test 3 is much more dependent upon the language habits the pupils developed in grades 1 to 8 than upon instruction received since they entered high school. The 11th and 12th grade performance on this test is more appreciably influenced by high school instruction in language, but all the averages may be more dependent on what happened to the students before they entered than since they entered high school."[1]

Since we are interested in the measurement of high school quality--and not that of the total educational system--a measure of gain must be developed:

> To evaluate the high school program alone, as distinct from the elementary and junior high school programs, one would need to know how much the pupils improved while they were in high school. To measure the effectiveness of the high school program as such, one should determine the gain in test performance for a typical class of students from the time of entrance to high school to the time of graduation (28, p. 21).

Consequently, the most reasonable measure of academic gain would be the difference between, say, the 12th grade composite score and the 10th grade score, for some "representative" class.

Such a longitudinal comparison, which is the best we can provide at present, is still full of pitfalls. First, "changes in the composition of a class through student transfers or school reorganization can have a marked effect on class averages." Second, "gains revealed by these comparisons are dependent upon many other factors besides effectiveness of instruction. Particularly important are:

1. The level of intelligence or of scholastic aptitude for the group.

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[1]See (28, p. 20). Test 3 is entitled, Correctness and Appropriateness of Expression.

2. The nature of the group's out-of-school environment and educational opportunities.

3. The extent to which the students are motivated to do their best on the tests at each of the two testings.

The effects of all possible factors should be investigated before judgments are made about the curriculum or the instructional program" (28, p. 22). In addition, other factors may influence the performance on these tests, such as:

a. On the part of the pupil: Motivation, temporary and permanent health, home environment, previous school experiences (especially if he is a transfer student).

b. On the part of the school: Curriculum, textbooks used, teaching materials supplied, general adequacy of school plant and equipment, type and extent of supervision, administrative policies, general harmony within school staff.

c. On the part of the community: Type (industrial or rural), population (foreign or native, heterogeneous or homogeneous), general level of culture, interest in educational matters, financial support of schools, cooperativeness toward school administration (21, p. 46).

Finally, if the undesirable practice of "coaching pupils specifically on items which the teacher thinks or knows will occur in the tests" takes place, then the tests' "validity as measures of general achievement and ability is gone (21, p. 59).

Chapter 2 surveys much of the literature on the economics of education, particularly the journal articles that were written in the last ten years. Of course, as many new publications appear constantly, and in growing numbers, the review does not cover those publications that have appeared since summer 1967. Nor do we pretend it to be exhaustive. In addition, interesting material included in the books by Schultz (35) and Becker (3) has been touched upon only when it pertained directly to the discussion at hand.

Next, Chapter 3 summarizes the arguments and empirical results that involve the use of the ITED scores, where the latter are hypothesized as being the accepted measure of output, or school quality. In the following chapter, the implications of possible economies of scale in high school operations--after an allowance is made for different levels of quality among schools--are studied both theoretically and empirically.

Last, Chapter 5 provides a new framework for the examination of high school quality. Given that the ITED scores do not provide us with an unequivocal measure of school quality, we choose, on *a priori* grounds, those factors which we believe affect quality. Further, these factors are closely scrutinized. On the basis of these "quality" factors, a new "quality index" is formulated.

CHAPTER TWO. A REVIEW OF THE LITERATURE

To the educator, to the school administrator, and to many laymen, the intrusion of economic analysis into the realm of education may seem unwarranted. In particular, the educator fears the examination by the economist of what C. E. Beeby (5) calls "the classroom conception of quality." That is, while the economist is acknowledged the right to examine the aspects of education "outside the classroom and into the market-place, where the quality of education is measured by its productivity," no such right is bestowed upon him where such items as the performance of students in the "three R's" or "the acquisition of a given range of facts about history, geography, hygiene and the like" are concerned (5, pp. 10-13). Moreover, since education has so many diverse aspects, such as cultural, sociological, psychological, spiritual and moral considerations, endeavors by economists to shed some light on educational policy are regarded with suspicion. Nevertheless, economists have recently embarked upon a wide range of studies dealing with educational systems.

It will be useful to classify the available literature into three categories. The first will encompass studies which relate to the economic value of education--to benefits, costs, rates of return and the like. The second, an emerging topic, includes studies relating to the manpower-planning approach to the study of educational planning. The third includes studies of the measurement of educational quality and the uses of such measurements.

The Economic Value of Education

It had been evident even to the classical economists that a full notion of capital must include human capital. As Kiker (24) points out, the value of humans was included in the definition of capital by such great economists as Sir William Petty, Adam Smith, Say, Senior, List, von Thunen, Walras and Fisher. Their main interest was in calculating the value of humans for the following purposes (24, p. 481):

1. To demonstrate the power of a nation.
2. To determine the economic effects of education, health investment and migration.
3. To propose tax schemes believed to be more equitable than existing ones.
4. To determine the total cost of war.
5. To awaken the public to the need for life and health conservation and the significance of the economic life of an individual to his family and country; and
6. To aid courts and compensation boards in making fair decisions in cases dealing with compensation for personal injury and death.

Two main approaches were used to calculate the value of humans. The first is called the cost of production approach, while the second is the capitalized value approach. Define C_x as the total cost of producing a human being (neglecting interest, depreciation and maintenance) through age x. Also, let c_o denote costs incurred up to the point of birth, k the annual percentage increase in cost, then Ernst Engel's formula is:[1]

$$(2\text{-}1) \qquad C_x = c_o \left[1 + x + k\left(x\,(x+1)\,/\,2\right)\right]$$

Theodore Wittstein formulated two additional methods of computing the cost of producing humans. His first formula follows the cost of production approach, while the second is a mixture of the former approach and the capitalized value one:

$$(2\text{-}2) \qquad C_n = a\,R_o \frac{L_o}{L_n} r^n - aR_n$$

$$(2\text{-}3) \qquad C_n = X\,R_N \frac{L_N}{NL_n}\, p^{N-n} - aR_n$$

where "a is the annual consumption expenditures including education for an average German male in a particular occupation, $r = (1 + i)$, where i is the market interest rate; $p = 1/r$; L_n is the number of men living at age n in a life table; R_n is the value of age n of a 1-thaler annuity (for a given r and purchased at birth); X is the value of the future output of an average man in a particular occupation; N is the age at which this man enters the labor force (24, p. 483).

The cost of production approach has been attacked on

[1]See the Appendix for the derivation of 2-1.

many grounds. Kiker objects to it because, in his view, there is no "simple and necessary relationship between the cost of producing an item and its economic value." Whether or not we agree with Kiker on this point, it seems that modern writers prefer the capitalized value approach. And while the following formulas were advanced by Dublin and Lotka in 1930,[1] these were originated by William Farr (though in a slightly different form) as early as 1853:

$$(2\text{-}4) \qquad V_o = \sum_{x=0}^{\infty} v^x P_x \, (y_x E_x - c_x)$$

$$(2\text{-}5) \qquad V_a = \frac{P_o}{P_a} \left[\sum_{x=a}^{\infty} v^{x-a} P_x \, (y_x E_x - c_x) \right]$$

$$(2\text{-}6) \qquad C_a = \frac{1}{P_a} \left[\sum_{x=0}^{a-1} v^{x-a} P_x \, (c_x - y_x E_x) \right]$$

where "V_o is the value of the individual at birth; $v^x = (1 + i)^{-x}$ is the present value of \$1.00 due x years later; P_x is the probability at birth of an individual living to age x; y_x is yearly earnings per individual from age x to x + 1...; c_x is the cost of living for an individual from age x to x + 1." Also, V_a is the value of the individual at age a. Finally, C_a is the cost of producing an individual up to age a,

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[1]For bibliography on this and other sources, see Kiker (24, pp. 497-498).

while E_x is the proportion of individuals employed from age x to age x + 1 (24, p. 484).

We shall concentrate our efforts in the remainder of this section on the refinements of the formulas presented above, as well as the discussion of the variables involved.

Problems in estimation

Even if we can devise a perfect technique for the evaluation of human capital, it will be of no avail if appropriate data do not exist. And while data concerning physical capital abound, few are available for the human counterpart.

In addition, given the available data, we are still confronted with a store of problems. In the first place, the nature of the data rarely fits exactly the purpose of the study. For example, the Census of Population contains some data on personal incomes cross-classified according to educational levels of the income recipients. Yet these are only cross-sectional data, and we would like to have longitudinal data showing the effects of different educational inputs upon incomes of members of the same cohort (e.g., all persons born in a given one-year or five-year period). Furthermore in such census reports, no account is taken of the ability and other background of the individuals in each income or educational category.

As most writers attempt to measure the value of an incremental education unit, for example, the value of college education, we need marginal income figures. However, Census and other data give us merely the mean and/or median income for the group. One obvious question is, therefore, Which of the two (mean or median) is most appropriate for our purposes? The answer depends on whether the mean is larger (or smaller) than the median and whether the average rate of return exceeds (or is smaller than) the marginal rate of return. Renshaw argues that the marginal rate of return is smaller than the average rate of return because of diminishing returns "and as a consequence of the likelihood that any general increase in educational attainment will be accompanied by a decrease in the average level of ability" (32, p. 322). Further, Renshaw contends that "median [income] differentials are smaller than mean differentials owing to the skewness in the distribution of income."[1] Hence, Renshaw argues, it appears that the median is more appropriate. Moreover, there are some practical considerations. "In the first place, census data are typically reported that way. Since the Census is the only comprehensive source of income data classified by education, one is almost forced to start with medians. Another reason is that the most recent Census definition of income includes property income as well as wage and salary income. Median income differentials are likely to be less

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[1]See Renshaw (32, p. 322) and Becker (3, pp. 136 ff.).

biased because of property income than are means" (32, p. 322). Concurring with Renshaw on this issue is D. S. Bridgman. In his view, the method which uses the median "eliminates the greater than proportional weight given in calculating means to a relatively limited number of quite large incomes" (9, p. 181).

On the other hand, it may be argued that dynamic changes "might act to maintain a constant marginal rate of return over time" in which case the mean is a more accurate representation of the marginal rate of return (32, p. 322). In addition, should we feel that the rates of return obtained by using the median are too low (the reasons for which will be explored later), the mean may be the preferred statistic.

Furthermore, while recent Census data did include property income, many sources of income, other than wages and salaries, have not been included. An example is dividend-income.

Consumption and investment in education

Professor Theodore W. Schultz (35) was one of the first to recognize that education involves not only investment in the human agent but also a certain amount of consumption. Thus, a student undertaking college education receives (1) a potentially higher income in the future, a result of his investment, and (2) an immediate (as well as future) reward in the satisfaction that he derives from his education. Inasmuch as the student acquires new tastes, the added satisfaction from utilizing these in the future constitutes investment for future consumption. The point is that most studies which have attempted to calculate the returns to education have had to ignore the consumption element, regardless of how important it may be.

Moreover, the consumption element discussed above should properly include a host of what are commonly called third-party effects, external economies (and diseconomies) or simply externalities. These include the satisfaction and benefits incurred by the individual's family and associates, his future employers, his neighbors, and the society as a whole. Burton A. Weisbrod (39) goes even farther than that. He shows that the investment component, as well, is grossly underestimated by the conventional methods (to be discussed shortly). We shall defer the discussion of these matters for a later stage.

Rates of return

It will be useful at this point to analyze the ways in which rates of return to education have been obtained in the literature. One such attempt was made by Becker (4), who was interested in the investment (or underinvestment) in college education. The technique used was to take Census income data classified by education, adjust for ability, race, unemployment and mortality. The costs of acquiring the education are subtracted from income. The costs of education are composed of foregone earnings, direct costs to the student (tuition, fees, books, etc.), or if we are interested in the social, rather than the private, cost, the difference between the total cost of providing the student with all the necessary facilities and his direct tuition costs. The remaining figure is, thus, the net revenue (income over cost) to the student. To get a rate of return, we must discount the stream of net revenue by some interest rate to arrive at a present value figure. Becker used this scheme to arrive at a rate of approximately 9 per cent for 1940 and 1950 Census data (including urban-whites only). In a later publication, the results were revised: 14.5 per cent for 1940, and 13 per cent for 1950 (5, p. 78). But the methods used in this latter study were more refined, incorporating into the analysis such factors as the secular rate of growth in earnings and tax rates which have not been considered previously.

Similar methods have been used by others to calculate various rates of return. Schultz (36), for example, gives an estimate of 14.3 per cent as the rate of return on four years of high school in the U.S. as of 1939. For 1958 he reports the following rates: elementary, 35 per cent; high school, 10 per cent; and college, 11 per cent. One more set of estimates is given by Lee Hansen (17). He shows that for males, in 1949, the marginal rate of return rises rapidly from the completion of the first 2 years to the completion of the 7th and 8th year of schooling, from a rate of about 9 to 29 per cent. The marginal rate of return then declines for high school and college; the 11th and 12th year of schooling show a return of nearly 14 per cent and the 15th and 16th year a strong 15 per cent. And Renshaw reports the following:

> In the Thirty-Eighth Annual Report of the National Bureau, Becker presents some preliminary estimates of the rate of return earned on income invested in a college education in 1940. "The rate of return was about 12 per cent on income invested by society, while it was over 14 per cent on that invested by the individuals and their families." In an unpublished paper, Telser arrives at about the same conclusions. "The internal rate of return of a college education is about 15 per cent" (32, pp. 318-19).

Using time-series data, Renshaw obtained similar figures on "the average productivity of education."

It must be emphasized once again that these rates are direct rates of return inasmuch as they do not consider external effects (whether positive or negative). Further, it is far from clear that the costs used in the computation of net income are indeed the "correct" costs. We shall examine both of the above qualifications in turn.

External or neighborhood effects

While Becker, in his 1960 article, had already paid lip service to the existence of "external economic and military effects," which, as he then contended, are brought about by a select group of students whose major fields are in the natural sciences--and only for those in high academic ranks--he failed, at that stage, to realize the importance of the consumption element as well as the various external effects. This failure was brought to light by Weisbrod (39) who lists these external effects in an almost exhaustive manner.

One argument is used in almost any introductory public finance textbook to justify the interference of government in the market, namely, the fact that the education of one's children will spill-over some benefits on his neighbors, his own family, and the community as a whole.[1] In the first place, an educated person's mode of behavior is likely to be better in terms of the norms of the society than that of the uneducated person. Also, such a person is more likely to participate in civic activities. The result may be a considerably more pleasant neighborhood. Weisbrod suggests that the value of such benefits may be gauged by "studying voting behavior on school issues among non-parents."

Second, the student's family stands to gain as well. When we measure the rate of return to elementary and high school education, we must note that mothers are free to go to work, if they so wish. One can measure the extra value to the mothers of this opportunity by calculating the amount

[1]See, for example, Buchanan (10, pp. 422-423).

that mothers would have to pay baby-sitters. Since many mothers would probably not go to work at all if they had to take care of their children, this estimate is likely to be biased downward. In any event, Weisbrod demonstrates that this gain is equivalent to about 25 per cent of elementary school costs.

Third, there are substantial gains to society, whether the employers and colleagues of the subject, the taxpayer, or society at large. Obviously, there are employment-related benefits. First, the employer stands to gain the more education his employees obtain. Further, the other employees, with whom the former student will associate, tend to gain, as well, the more education he obtains: the productivity of one employee depends on that of the others. So everyone has "a financial interest in the education of his fellow worker."

As suggested above, the taxpayer benefits in the form of lower law enforcement costs (perhaps also lower insurance rates, etc.), as we expect less crime to originate from the more educated citizenry. It would be an interesting exercise to test the above hypothesis on the basis of police records.

Finally, as Weisbrod suggests, society in general stands to gain from more education. For example, the more people obtain literacy, the more the demand for books, checking accounts, etc. Then mass production and distribution techniques may be applied so that the price of the above may be quite low. Also, the more people are engaged in research, the more the benefits to society in the form of inventions and innovations for which the inventor cannot generally collect all the fruits of his labor.

Other direct benefits

In addition to the neglect of external effects in the analysis of educational returns, there are a number of direct returns which have not been considered. We shall discuss the options or opportunities that education opens to the student. One such option, what Weisbrod calls "financial option," underlies the fact that schooling gives the student the opportunity to undertake more schooling (and, according to Mincer (30), more on-the-job training as well). He therefore proposes the following formula for measuring the rate of return to education:

$$R_j = R_j^* + \sum_{a=k}^{z} (R_j^* - \overline{R}) \frac{C_a}{C_j} \cdot P_a \tag{2-7}$$

where R_j^* is the rate of return at year j computed by the usual method (i.e., "it is the difference between the present value of expected future earnings of a person who has attained, but not exceeded, level j, and the present value of expected future earnings of a person without education j"). $\overline{R}$ is the alternative rate of return on "the next best investment opportunity;" C_a is the marginal social cost of obtaining the incremental education a, and P_a is the probability that a person with educational level j will undertake level a. Using the data supplied by Schultz (34), Weisbrod demonstrates that, by using a discount rate of 5 per cent (i.e., $\bar{R} = 5\%$), the rate of return (R_j) for high school education increases by at least 2.8 per cent, while that of elementary education increases by 12.3 per cent!

The second class of options is Weisbrod's "non-financial options." For instance, a college professor has many non-financial advantages. The monetary value of such options can be measured by the difference between the wages and salaries that he could have earned in alternative employments and that which he actually earns. Another example is what Weisbrod has called the "hedging option." That is, education, particularly a general one, enables a person to change jobs more easily. Moreover, by acquiring knowledge, an individual is able to perform a number of services himself that would have otherwise been performed in the market (this is Weisbrod's "non-market option"). An example of this is the filing of income-tax returns. Weisbrod claims that the savings to the total populace by filing their own tax-returns is about 0.8 per cent of total elementary school costs. Other examples are typewriting and driving.

Intergeneration effects

Not only do the analyses of the rates of return to education neglect a host of direct and indirect returns to the individual and his contemporaries, they also fail to take into account alleged intergeneration effects (38). It has been shown that a strong correlation exists between the educational level of the parents and the likelihood that their children will embark on additional educational training as well. Therefore, if the children's income will be enhanced by their extra education, and if this extra education was obtained because the parents decided to spend more on education, it follows that part of the children's return is (indirectly) attributed to the parental educational expenditure. In sum, if we confine ourselves to the head of the family only, the head's rate of return on education is likely to be much larger than previously envisaged.

Swift and Weisbrod (38) propose the following empirical model. Let C_y denote "the cost of each of the k years of the head's education," $\overline{R}$ "the annual return on the next most profitable investment," and R^* "the gross return on the second generation investment under consideration." Then we want to solve for r--the rate of return on "the additional incentive which the parent's schooling apparently provided to the child"--in Equation 2-8:

$$(2\text{-}8) \qquad C_y\,(1 + r)^y = \sum_{t=n+25-a}^{90-a} \frac{R^* - \overline{R}}{(1 + r)^t}$$

where a is the age at which the head terminated his education, and n is the age at which the child began to work. Further, a number of assumptions are made. First, it is assumed that the head did not become a parent until he was 25 years old. Second, we make specific assumptions as to the value of $\bar{R}$. In addition, many other assumptions have been made in the process, so that the authors warn us that the results "should be regarded as tentative at best." In any event, if we accept the model and the assumptions, it appears that at least the cost of the parents' high school education was paid back by the intergeneration effect. Although this conclusion does not hold true for all incremental education levels chosen by the parents, at least part of the parent's cost of any incremental education unit was paid back to the children.[1] Moreover, it is the concept, rather than the particular estimates, that is of significance; the latter can and should be improved at a later date.

School costs and earnings foregone

So far we have used the term "cost of education" without much elaboration. However, it is far from obvious what to include in this term and how to include it. Perhaps the most straightforward portion of the costs of schooling is the direct cost (to the individual or to society). This will

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[1]The specific results are reported in (38, Table 2, p. 647).

include expenditures on fees, books, and the like. Also, for the measurement of the social cost we can include in this category most of the expenditures by the school under question. However, in the latter case, school expenditures do not always indicate the exact cost of schooling. First, expenditures may include such items as building improvement, or other capital outlays, which do not necessarily represent school costs for any particular year. Second, public school budgets do not include costs incurred by the students or their families. For example, a student may be required to buy his own school supplies (paper, pencils, and miscellaneous equipment). Or his family may be required to provide him with transportation to and from school. Further, a more rigorous analysis should not neglect the fact that public schools (and other educational institutions) are exempt from tax payments. This fact implies that society must face an additional implicit cost in its educational enterprise, one that should properly have been added to other explicit school costs.

More important yet is the major portion of school costs--earnings foregone. To illustrate the importance of these costs, I reproduce the figures given by Schultz in Table 2-1 below for four countries. The method used by Schultz to arrive at the U.S. figures assumes that students forego 40 weeks of earnings annually. Their weekly foregone income is measured by the corresponding income figures for workers in

Table 2-1. School costs, earnings foregone, and total costs of schooling per student per year in the United States, Israel, Mexico and Venezuela[a]

	School costs	Earnings foregone	Total	Earnings foregone as per cent of total cost
United States, 1956 (dollars)				
8 years elementary	280	0	280	0
4 years high school	568	852	1,420	60
4 years college	1,353	1,947	3,300	59
Israel, 1957-58 (Israeli pounds)				
8 years elementary	140	30	170	18
4 years high school	670	1,000	1,670	60
3 years higher education	2,481	2,930	5,411	54
Mexico, 1957 (pesos)				
6 years primary	360	0	360	0
6 years secondary	1,794	2,833	4,627	61
3 years university	2,426	3,280	5,706	57
Venezuela, 1957-58 (bolivars)				
6 years primary	400	0	400	0
5 years secondary	1,200	5,000	6,200	81
4 years university	5,000	12,000	17,000	71

[a]Adapted from Theodore W. Schultz (35, Table 1, p. 29).

a comparable age group. Multiplying the weekly wage by 40 we get the total annual earnings foregone. Schultz concludes that high school students forego the equivalent of about 11 weeks and college and university students about 25 weeks of average manufacturing earnings. Compared to total school costs, high school foregone earnings were about 73 per cent in 1900 and 60 per cent in 1956. College and university foregone earnings were about 59 per cent of total costs in 1956. No earnings foregone were assigned to elementary students, even though there are strong indications that in 1900 or thereabout they were quite substantial (one third of the population was on farms).[1]

Internal rate of return or present value

The presentation of rates of return to education presupposed the policy implications embodied in these rates. Thus, the "internal rate of return rule" suggests that we compare the rate of return (as explained above) to, say, college education to the rate of return earned on the best alternative investment. Then, if the latter exceeds the former, it would not be worthwhile to undertake (or support) the investment in college education--and vice versa. In a path-breaking article, Jack Hirshleifer (20) shows that "the contention of those who reject the internal rate of return

[1]See Schultz (34).

as an investment criterion" are on the whole justified. Further, while the present value rule "is at best only a partial indicator of optimal investments and, in fact, under some conditions, gives an incorrect result," Hirshleifer's study "provides some support for the use of the present-value rule" (20, p. 329). The present-value rule simply states that we should compute the present value of <u>all</u> alternative investment projects, and choose that which has the highest present value.

There are also some practical reasons for the use of the latter rule. As Wilkinson observes, it is easier "to calculate the present value of each project than it is to subtract one income stream from another and compute rates of return for <u>every</u> possible comparison of projects" (43). However, this observation makes sense only when we attempt to compare the returns to one occupational group, say, with those of another. But in such studies as reported above, the internal rate of return rule is the simpler technique. Also, under certain conditions, the two rules yield identical results (20, p. 333).

<u>International comparisons</u>

So far we have limited ourselves to American education. Many data are also available on other countries, but here we shall report just a few observations.

Bruce Wilkinson has studied some aspects of human capital in Canada. A very interesting question is whether people would change jobs if the return in alternative employments were greater. He thus compared the present value (to be denoted henceforth p.v.) of income for different levels of education for selected occupations. Using discount rates of 5, 8 and 10 per cent, he found that, in general, increments in education result in p.v. increments as well--except for two years of college at 8 and 10 per cent. However, for different occupations there are different results. For example, for some occupations, such as typesetters and draftsmen, four years of high school may not be worthwhile--even if the discount rate were only 5 per cent. And to illustrate the way in which marginal differences in p.v. cause mobility between occupations, he presents his study of "changes in discounted returns to teachers and engineers in relation to changes in college enrollment." Thus, between 1957 and 1961 the following observations were made:

a. teachers' p.v. increased between 17 and 20 per cent;

b. enrollment in education increased 133 per cent;

c. engineers' p.v. increased between 4 and 5 per cent;

d. enrollment in engineering increased by 3.8 per cent.

A partial explanation of (a) and (b), according to Wilkinson, includes the following:

1. "The increasing numbers of women attending university frequently favor education;"

2. "teacher training has been shifted from teachers' colleges to the university campus and such colleges have been incorporated in education faculties;" and

3. "the increasing number of students attending university may choose education because it is easier to finance than engineering: a person can take on two years of training, then commence teaching and obtain the balance of his university education at summer school or by correspondence courses designed for this purpose."

Nonetheless, there is still a significant relationship between results (a) and (b) pointing to the fact that some tendency exists to move to (or to choose) one occupation rather than another when changes in the present value of earnings occur.

Another study, this time on India, was undertaken by V. N. Kothari (26). The purpose of the study was to "measure the magnitude of resources used up by education" in India. Some of the peculiar features of the Indian economic and educational system had to be taken into account in estimating some of the costs incurred by pupils in India. For example, private tutoring is quite important. Kothari presents two methods for the estimation of earnings foregone. In both cases, assumptions are made with respect to the "earning equivalents" that students could potentially obtain according to their age, sex and educational level. For example, high school students, aged 15 and above, in rural and urban areas are assumed to have the earning capacity of a primary school teacher. The two methods differ in that the so-called "lower" estimate (of earnings foregone) excludes altogether potential earnings by primary school pupils, reduces the number of potentially economically active female students in general education to 25 per cent (50 per cent in the "upper" estimate), and deflates the obtained (lower) estimate by 25 per cent to account for unemployment (instead of 10 per cent in the "upper" estimate).

Total educational costs in India reflect explicit and implicit (earnings foregone, in particular) costs. Since we have two estimates of earnings foregone, there will be two corresponding estimates for total educational costs. Further, in the "lower" estimate (of total costs) the estimated expenditures on private tuition is reduced by half. Nevertheless, the major difference between the upper and lower estimates of total educational costs in India is attributable to the differences in the estimation of earnings foregone.

Although the methods used to arrive at total cost figures are not equivalent to those used by Schultz and others in the preparation of the American data, some useful comparisons emerge:

1. Earnings foregone are a very important component of total resource costs in Indian education--between 45 and 55 per cent of total costs.

2. Total educational costs as a proportion of NNP increased from 3.6 per cent in 1950-51 to 6.5 per cent in 1959-60.

3. Total educational cost as a proportion of total net investment = .60 for the entire period. That is, educational and physical capitals (in India) have been increasing at about the same rate.

4. While, according to Schultz (34), total educational expenditures in the U.S. were about 34 per cent as large as total gross investment in physical capital in the U.S., total costs of education in India were between 27 per cent and 35 per cent (for the lower and upper estimates, respectively) of gross investment in India.

5. Further, while cost of schooling per pupil in India is far less than that in the U.S., the ratio of per pupil cost to per capita national income is much lower in the U.S. than it is in India. In other words, the burden of the educational enterprise is greater in India than it is in the U.S.

6. Finally, "the higher stages of education are relatively costlier in India than in the U.S.--whether measured in terms of cost of primary education or measured in terms of per capita income."

Another study examines the profitability of education in Israel (25). The results of Mrs. Klinov-Malul's investigation show that for the most part only primary education is profitable, both to the individual and to the State. Secondary education is not, for two reasons. First, sêcondary school fees are high (assuming the parent pays full fees)--

higher even than university tuition fees. Second, in Israel's egalitarian society high school graduates do not earn much more than primary school graduates. And as far as college and university education is concerned, it is only slightly more profitable to the individual, whereas it entails some loss to society--the equation varying according to occupations. Of the four occupations studied, lawyers make no profit, engineers and accountants do make profit, while medicine brings no profit to the individual and a substantial loss to society (it must be remembered that external and other benefits are not included, whereas most costs are).

Finally, studies by Blaug and others on Great Britain are revealing, but a completely independent survey must be undertaken in order to cover such a wide field.[1]

The stock of human capital

In this final part of this section we want to analyze the total value in the economy of human capital, as well as the average value of individuals of given age and educational levels. Such attempts have been made by several authors. Weisbrod, for example, measured the average value of a human being in the United States, as well as the total stock of human capital (40). If, indeed, we could have a measure of a person's value, given his age, on the basis of some "average" figure, it would be of great significance in settling court

[1]For some references, see (12).

cases arising from accidental (or otherwise) death or injury, in determining the optimal amount of life insurance to be taken each year, and in the formulation of immigration policies--to mention but a few applications. Weisbrod's formulation is thus:

$$(2\text{-}9) \qquad V_a = \sum_{n=a}^{\infty} \left[Y_n \, P_a^n \frac{1}{(1 + r)^{n-a}} \right]$$

where V_a = present value of expected future earnings; Y_n = value of productivity of a person at age n; and P_a^n = probability that a person of age a being alive at age n.

Weisbrod uses this formula with two different values for r (the rate of discount), 4 and 10 per cent. Incorporating earning figures of the Census Bureau, he arrives at income-age profiles with the following conclusions:

1. V_a is positive even for ages 0-4. This implies that excess of income over consumption is much higher in subsequent years than excess of consumption over income in early years. Now, Y_n is gross productivity, which is taken to be the person's income. A net productivity concept is arrived at by subtracting a "consumption" component from Y_n.

He defines a "person's net contribution to 'society'" by the difference between the marginal consumption "associated with a change in family size" and gross value of output. The former concept is measured by considering families whose "heads" are of various age and income groups, so that one can measure extra consumption associated with the addition of an individual of a given age to a particular family (having so many children of such age and sex with income Y and age X, etc.).

2. V_a reaches maximum when a is approximately 30--sometimes termed "prime of life" (20, p. 431). At this age, V_a is between \$20,000 and \$30,000, implying a very high value of an "average" person.

3. On the average (over one's lifetime), a person's worth is over \$13,000 using net Y_n and a 10 per cent discount rate, and if we use $r = 4\%$, and a gross value for Y_n we get an average value of \$33,000. The results apparently indicate that additional humans in the U.S. will, on the average, add to economic growth. This conclusion, however, may not be true for other countries. In fact, there are indications that the value of additional humans in India, once more on the average, is zero or even negative (40, p. 433).

4. Total human capital in the U.S. for 1950 is estimated at \$1,335 billion (for $r = 10\%$) and \$2,752 billion (for $r = 4\%$), compared to \$881 billion for non-human capital in 1949. Corresponding net values were \$1,055 billion and \$2,218 billion for $r = 10\%$ and $r = 4\%$ respectively. The implications that one might draw from these results are that more emphasis should be placed on human capital in the form of its maintenance and growth in the fields of education, health and retraining.

Another set of estimates is provided by Schultz, who uses a different method. His approach "rests on estimates of the investment in schooling in people who are in the labor force and the rate of return earned on this investment. The first, expressed as a stock of capital in 1956 dollars, came to $180 billion for 1930 and $535 billion for 1957 (35, p. 45). And while Schultz's figures are substantially below those of Weisbrod, the identical oonclusions stand.

A cost-of-production procedure is employed by Renshaw to arrive at one more set of estimates. Utilizing figures supplied by Schultz in an unpublished paper (37), estimates of total earnings and expenditures for high school and college and university education are converted into 1950 prices. Then, by "summing the figures from 1900 to 1950, one can obtain a rough estimate of the stock of educational capital based on cost of production: $241.7 billion." With the use of some simplifying assumptions, Renshaw "endeavored to arrive at a stock figure by capitalizing median income differentials existing in 1949. Discounted at a five per cent rate, the present value of these differentials amounts to $329.9 billion; at ten per cent, $201.4 billion" (32, p. 322).

Undoubtedly there are many conceptual problems with the above estimates. As Weisbrod himself notes, "the estimates can be improved and extended. The benefits of doing so appear to make the costs worthwhile" (40, p. 436). Nevertheless, all of the estimates point to the fact that the stock of educational capital in the U.S.--however defined--is of a substantial magnitude.

Manpower and Educational Planning

The idea of manpower planning has captured a wide audience in recent years, and as education constitutes one of the prerequisites for the creation of a modern labor force, it, too, has been included in such schemes. One of the justifications for such planning has already, though only implicitly, been elaborated upon, namely, that the external costs and benefits of education prevent the formation of a "rational calculus" for educational supply and demand by both educational institutions and students (respectively). In two interesting articles, Blaug (7, 8) attempts to analyze this and other arguments for and against the use of the manpower-planning approach to educational policy-making as opposed to what Blaug calls the anti-manpower planning approach (the one which relies on market forces to determine the extent of education undertaken by the populace).

Suppose that a meaningful "rational calculus" can be ascribed to students (or their parents) in choosing amounts of education (as well as fields of education). If r_e is the rate of return to a particular type of education, while r_b is the best alternative investment opportunity rate, then, according to Blaug (7), we can expect that the demand for that particular type of education will vary directly with r_e and inversely with r_b. Further, if suppliers of education are similarly flexible, and if the "price of education" is defined as r_b/r_e, then it is expected that supply and demand will lead to an equilibrium point at $p = 1$, where the costs of education exactly match the benefits from that education. If we accept a strict anti-manpower planning approach, market forces will insure that such an equilibrium will actually be reached (sooner or later). Also, when we speak of costs and benefits, the divergences between the social and private counterparts of these will not be very significant. On the other hand, a strict manpower planning advocate would reject the possibility of such an equilibrium, arguing that a "rational calculus" cannot be assumed--i.e., students choose increments of education as well as areas of study not at all according to costs and returns, but rather according to other non-economic principles (such as social prestige). Blaug (7, pp. 170-171) contends that in Britain an excess demand for education exists (point R in Figure 2-1), and that only a change in the supply of university education can alter the situation (which will yield a "non-equilibrium" price where $r_b \neq r_e$). Blaug himself takes a middle-of-the-ground approach (7, p. 182):

> If there is anything to the idea of a rational educational calculus, enrollment projections that ignore earnings patterns in labour markets, and thus neglect the price-elasticity of demand for education, are

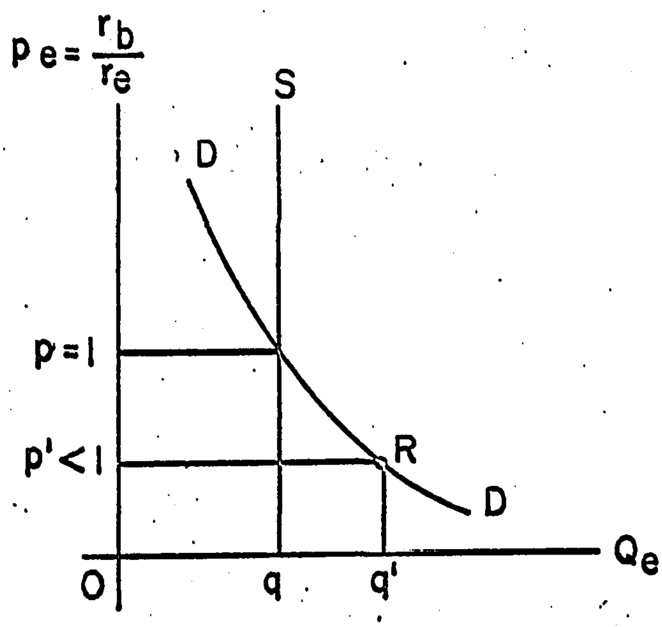

Figure 2-1. Demand and supply in an educational market

> almost certain to go wrong. Likewise, manpower forecasts that indicate the minimum numbers of specially qualified people that will be required if certain targets for economic growth are to be realized are very likely to be misleading unless it is possible to control the output of education by fields of specialization and, furthermore, to absorb the additional supply of educated people into employment without radical changes in earnings differentials; every radical change in earnings differentials will alter the demand of industry for these people and the demand for like-minded people to acquire that sort of qualification. In short, the interdependence between the "market" for extra education and the market for educated people makes it impossible to discuss either without reference to the other.

To illustrate this interdependence, a 4-quadrant chart (Figure 2-2) is used by Blaug (20, p. 172). In the first quadrant, we have Figure 2-1 above. In the one below it, the demand for educated people as a function of the starting wage rate is sketched. To the left, in the third quadrant, age-earnings profiles for different educational levels of the same profession (in this instance, technology graduates) are shown, and in the second quadrant we have the present values of future earnings streams for the given professions. The latter are derived as follows: Suppose that all of the costs of education are incurred in a lump sum fashion, and denote these by C. Further, suppose that only a certain portion, α , of the earnings differentials can be attributed to education (the other portion is attributed to ability, socio-economic conditions, etc.). Then, if E_t denotes the earnings differentials in year t, the rate of return, r_e is determined by:

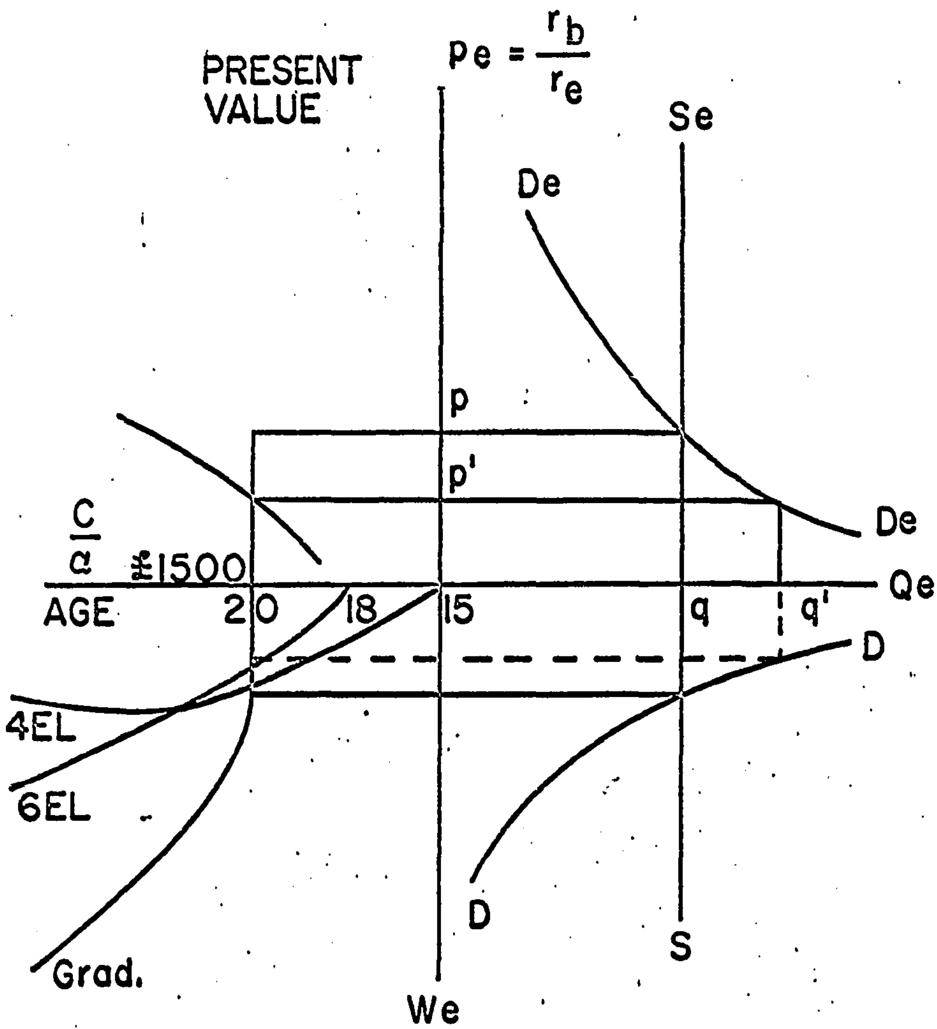

Figure 2-2. Interdependence between the market for education, the market for educated people, age-earning profile and present values--adapted from Blaug (7, p. 172)

$$\text{(2-10)} \quad \text{Present value} = \sum_{t=0}^{43} \frac{E_t}{(1 + r_e)^t} = \frac{C}{\alpha}$$

Hence, in quadrant 2, we can depict the relationship between r_e and C/α (8, pp. 252-261).

Some models of educational planning

In a forthcoming paper, Fox and Sengupta (15) comprehensively analyze a number of models of educational planning. Therefore, I shall be very brief in my comments. First, linear programming models, which may include only a part of the total economy, or the economy as a whole with education as a separate sector--once more with many variations which are discussed by Fox and Sengupta--have been employed. For example, Irma Adelman (1) attempts to build an optimization model for investment in formal education and in its optimal allocation simultaneously. The model thus combines the "manpower requirement" and "cost-benefit" (or rate of return) approaches to the study of educational planning. A linear programming model for 4 periods of 5 years each is used with data partially characterizing the Argentine economy. The rest of the economy is disaggregated into just nine sectors, and three different objective functions are used:

a. Maximize the discounted value of GNP;

b. Maximize the growth rate of the economy;

c. Minimize the discounted sum of net foreign capital inflow.

The constraints to the program involve, among others, the structure of the Argentine educational system.

While Adelman's model is quite instructive in so far as model-building is concerned, her specific results should be viewed with much skepticism--she herself warns us that the results should not be used indiscriminately. For example, one outcome of the model is that only university graduates or drop-outs should be trained. Another is that commercial and vocational schools are not "utilized in the optimal school network." It seems that such results were obtained primarily because it was assumed by Adelman that the productivity of university graduates is 3-1/2 times that of secondary school graduates--which, as Bowles notes in the "Comment" to Adelman's article, is rather unlikely to be true.

Another model has as its purpose the provision of a "preliminary model that accomplishes" two objectives: (1) to develop "a method for projecting future labor requirements," and (2) to relate "these requirements to the output of the educational system" (11). Chance uses a Leontief system for labor-skills demand and a Markov-chain device for the supply of the various labor skills as a function of the educational system; he employs a number of simplifying assumptions to make the analysis manageable. In sum, a model is developed that will bring about a full-employment solution for the economy, via the use of the projections contemplated above. Chance also attempts to assess the economic value of the new solution in comparison to sub-optimal solutions. Finally, a variety of other models have been developed, many of which are discussed by Fox and Sengupta (15).

The Quality of Education

Basically, two methods have been used to measure educational quality. The first--and perhaps the more widely used--utilizes a priori beliefs, as well as subjective judgement concerning the choice of factors that are believed to affect school quality. The second, and the more objective of the two, purports to obtain a measure of school quality from test results which are objectively and impartially administered to a large number of pupils over time and space.

The subjective (first) approach is the easier one to use, but it entails a more difficult rationalization for its use on the part of its user. Hirsch, for example, built an index of "scope and quality" which is based upon what in his view, "many educators maintain..." (19, p. 31). While Hirsch is primarily concerned with primary and secondary schools, his quality index could be modified and extended into any type of schooling. Incidentally, we shall confine our attention in this section to sub-university schooling. Hirsch presents, first, an "ideal" model in which the following variables are included:

1. Class size. The implicit hypothesis is that the smaller the class size the better the scope and quality. To

measure class size one may use the average pupil-teacher ratio, using as the number of students enrolled the average daily attendance (hereafter to be denoted ADA), rather than the number of registered pupils. However, such a measure may conceal much detail. Hirsch proposes, therefore, an additional measure, that is, whether the school will offer a course in mathematics or a foreign language if only 10 or 15 students enrolled.

2. Grouping: "many educators maintain that good education requires that, within limits, students of common ability and interest be grouped together." A specific quantitative measure of this factor is not discussed.

3. Quality of the teaching staff:

   a. the per cent of experienced teachers;

   b. the background of the teaching staff--college training;

   c. "the method used for selecting new teachers and of appraising the quality of the existing staff;"

   d. teaching load; and

   e. the number and the variety of specialists included (19, p. 32).

4. Quality of school administration. "The leadership offered and ability of the school superintendent and his principals cannot be neglected; yet it is most difficult to appraise their contribution. Usually, principals who are not relieved from teaching cannot do a superior job."

5. Teaching program: Is there a good college prepara-

tory program? "How far does the Mathematics program go?" Similar questions may be asked.

While these items should, ideally, be put in the index of scope and quality, for practical reasons Hirsch drops entirely factors (2) and (4). Further, other simplifications are made. All in all, we have the following model:

$$(2\text{-}11) \qquad Q = g\,(A, B, C, D, E, F)$$

where A = the number of teachers per 100 pupils in ADA, B = the number of college hours of the average teacher, C = average teacher salary, D = per cent of teachers with more than ten years of experience, E = the number of high school credit units, and F = per cent of high school seniors entering college.

Given that the factors A, B, ..., F, were arbitrarily chosen, a weighting problem exists. To overcome this, Hirsch proposes equal weighting. This is justified as follows. First, "with six components to the index, the weighting system is no longer so very crucial. Doubling the weights of any one or two of the components will not greatly affect the magnitude of the index number." Further, a subjective test was carried on by Hirsch, in which the opinions of educators in the St. Louis area (in which the model was applied) were solicited as to the relative quality of the schools in the area. The rankings by the educators, "compared with the scope and quality index data, showed very close consistency (19, pp. 34-35).

Another, and similar, model for measurement of school quality was constructed by Riew (33). In his model, the following variables were used:

X_3 = average teacher's salary;

X_4 = number of credit-units offered (a two-semester course meeting five times a week is counted as one unit);

X_5 = average number of courses taught per teacher.

While data for other variables were also available, Riew omitted a number of them because they were strongly correlated with one of the above. Also, he chose to omit class size from his quality formula as there exists considerable controversy on whether or not this variable is of much consequence insofar as high school quality is concerned.

Another model is provided by Welch (41). His purpose is "to derive an estimate of the return to schooling from income data" which necessitates the adjustment of incomes "for differences in variables which may otherwise introduce bias" (41, p. 380). In his view, the return to education can be defined by "the number of units of schooling multiplied by the product of quality of schooling and the value of the marginal product of education." As a proxy for the quality component he uses a quality index, Q, as follows:

$$(2\text{-}12) \qquad Q = BZ_1^{z_1} \cdot Z_2^{z_2} \cdot \ldots \cdot Z_n^{z_n}$$

The Z's are the various school inputs which, supposedly, affect quality. The particular inputs used in his models (separately or simultaneously) are:

Z_1: "total current expenditure per pupil in attendance;"

Z_2: "average salary per member of instructorial staff;"

Z_3: "members of staff per 100 pupils;" and

Z_4: "enrollment per secondary school" (41, Table 4, p. 390).

It will be noted that this set of inputs contains expenditure and enrollment figures as quality variables, whereas such variables were treated differently in the studies reported above. Further, "the original observations are for 57 'states,' 10 southern states being designated as 2 'states,' one consisting of white persons and the other of nonwhites" (41, p. 379). The use of a whole state--or a certain population segment thereof--as the unit of observation for the determination of school quality is far from satisfactory as long as control over school quality resides, for the most part, in the local school board. Nevertheless, Welch's study opens new frontiers in the economics of education concerning the introduction of the concept of the total production of schooling (as defined above) and the ways in which that concept can (empirically) be estimated.

So far we presented attempts to define the quality of education on the basis of school inputs chosen *a priori*. It would be desirable to find an "objective" measure of quality, perhaps one that is based upon achievement tests of some sort. One such attempt has been made by Herbert Kiesling (23), although his purpose was not to measure quality but rather to assess the efficiency of school districts in the state of New York. In any event, an "expenditure model" of type 2-13 is formulated where

$$(2\text{-}13) \qquad Y = F(a, b, c, d, \ldots, E, u)$$

the lower case letters represent school inputs, E is expenditures per pupil, u is a stochastic (or an error) term, and Y is total school output.

Y, the output measure, is an average achievement score in a composite of standard subjects (based, for the most part, on the Iowa Test of Basic Skills). More interesting from our point of view is his "factor model" as in 2-14

$$(2\text{-}14) \qquad Y = g(a, b, c, \ldots, n)$$

2-14 is essentially a quality index of the "objective" type. Kiesling does not attempt to estimate Y on the basis of the inputs, a, b, c, etc., taken simultaneously. Rather he studies the effect of each of the inputs (under consideration) taken separately upon the quality measure (Y). In any event, this is a study in which "output" and "quality" are defined not on *a priori* grounds but rather on the basis of some "objective" criteria. We shall make an attempt, in the next chapter, to define quality in a similar manner--although the empirical results, in our case, are not very promising.

CHAPTER THREE. SOME QUALITY MODELS

Model Building: Descriptive vs. Operational Models

An historian may be content to describe or explain past occurrences on a verbal level. An economist cannot, however, satisfy himself with such descriptions or explanations. It is usually necessary, at some point, to make predictions, and these may or may not rest upon projections made from a descriptive model.

We may illustrate the point by considering models of educational content. Let Y_i denote quality of school i (of n schools) and X_{ij} the jth (of a total of m) input or factor used by the ith school. Then, a multiple regression model may be developed, which will estimate the historical contribution of each of the X_{ij} inputs to the quality Y. Thus

$$(3\text{-}1) \qquad Y = a + \sum_j b_j X_j \qquad (j = 1, 2, \ldots, m)$$

where Y is the estimated index of school quality.

As far as description is concerned, we may have reached our goal. It can now be shown that, say, k of the m variables X_j, $k \leq m$, are statistically significant. In other words, the k factors have, supposedly, exerted some appreciable influence in the "shaping" of the quality of school i.

A school administrator is likely, however, to be interested in the following problem: How can one maximize the quality of the school, given a certain budget constraint?

Suppose, then, that a budget of B dollars is available, and that factor X_j has a market price of p_j dollars per unit of X_j. Assuming linear relations between quality (Y) and each input, we then have the following problem:

$$\text{(3-2)} \qquad \text{maximize} \quad Y = \sum_{j=1}^{m} a_j X_j \, , \qquad j = 1, 2, \ldots, m$$

$$\text{subject to} \qquad \sum_{j=1}^{m} p_j X_j = B_o$$

The classical procedure in analyzing such a problem is familiar.[1] Form

$$W = \sum_{j=1}^{m} a_j X_j - \lambda \left(\sum_{j=1}^{m} p_j X_j - B_o \right)$$

We now want to maximize W. Therefore, we take the following partial derivatives and set them equal to zero.

$$\text{(3-3)} \qquad \frac{\partial W}{\partial X_j} = a_j - \lambda p_j = 0 \, , \qquad j = 1, 2, \ldots, m$$

$$\frac{\partial W}{\partial \lambda} = \sum_{j=1}^{m} p_j X_j - B_o = 0$$

Equations 3-3 form the 1st-order conditions, which state:

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[1]See, e.g., Henderson and Quandt (18, pp. 49-51).

1. for all

$$j \neq k, \quad \frac{a_j}{a_k} = \frac{p_j}{p_k}$$

In words, the (marginal) rate of substitution between factors j and k in the "production" of quality must be equal to the price ratio.

2. for all

$$j \neq k, \quad \frac{a_j}{p_j} = \frac{a_k}{p_k} = \lambda$$

That is, an extra dollar spent on X_j should increase quality by the same amount that an extra dollar spent on k would generate.

A number of difficulties arise in connection with this procedure:

1. The budget (and any other) constraint must be stated in terms of an equality, while often an inequality, such as

$$\sum_j p_j X_j \leq B_o, \qquad j = 1, 2, \ldots, m$$

may be called for.

2. If conditions of continuity as well as differentiability hold, we are assured of only a relative maximum. There could yet be another maximum, called maximum maximorum, in which school quality may be found to be greater yet--with the same budget constraint.

3. Finally, due to "corner solutions," kinks and discontinuities in the respective functions, the classical procedure may not be operational at all.

Fortunately, the field of mathematical, and in particular linear, programming has opened many more possibilities for the analysis of operational models. Thus, problem 3-2 may be restated in terms of a simple linear program:

$$(3\text{-}4) \qquad \text{maximize} \quad f(x) = \sum_j c_j X_j, \qquad j = 1, 2, \ldots, m$$

$$\text{subject to} \quad \sum_j a_j X_j \leq B_0$$

An optimal solution to program 3-4, if such exists, would indicate to the school administrator the optimal intensity of factor use (if quality is to be maximized subject to staying within the limits set by the budget).

Moreover, one can include in problem 3-4 not just one constraint, but as many as m (though not more than m). For example, in addition to the budget constraint, it may be specified that all teachers have at least four years of training, that the student-teacher ratio shall not exceed 35, that the per student value of building and equipment shall not be less than $1,000, and that the total number of units offered shall not be less than 25. For this particular example, the problem will be as follows:

$$\text{maximize} \quad f(x) = \sum_{j=1}^{m} c_j X_j \,, \qquad j = 1, 2, \ldots, m$$

subject to

$$\begin{array}{llr}
a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + a_{14}X_4 + a_{15}X_5 + \ldots + a_{1m}X_m & \leq & B_o \\
a_{21}X_1 & \leq & 4 \\
\quad a_{32}X_2 & \leq & 35 \\
\qquad a_{43}X_3 & \leq & 1{,}000 \\
\qquad\quad a_{54}X_4 & \leq & 25
\end{array}$$

One must note, however, that, first, a non-degenerate solution to the linear program (i.e., a solution which satisfies the constraints yet contains $X_j \neq 0$ for some j in the solution vector) may not exist at all. Also, even if an optimal solution is found, there may be another solution which yields the same value of the objective function. In other words, the uniqueness of the optimal solution, if such is found, cannot be assured. In any event, a considerable gain in insight can certainly be had from the use of linear programming models.[1]

Returning to the educational problem referred to above, it is necessary, first and foremost, to specify appropriate

[1]My source is Ladd (27).

values of the coefficients, c_j, in the objective function

$$f(x) = \sum_j c_j X_j$$

Further, as the analysis of Chapter 2 indicates, it is necessary, for a wide field of applications (see also Chapter 4) to establish a numerical index of quality.

This leads to the empirical formulation of the quality models of Chapter 3.

The Empirical Models

The basic model under consideration is:

$$(3\text{-}5) \qquad Y = f\,(X_3, X_4, \ldots, X_n)$$

where Y, in general, denotes an index of school quality, and the X's represent the various inputs of the school system. The variables which we shall use in subsequent models are defined below:

Y_1 = average composite score on the ITED (Iowa Tests of Educational Development) for the 12th grade (class of 1963)

Y_2 = difference between the average composite ITED score in the 12th grade and the average composite ITED score in the 10th grade

X_2 = total expenditure per pupil in average daily attendance, ADA

X_3 = average number of college semester hours per high school teaching assignment

X_4 = average number of different subject matter assignments per high school teacher

X_5 = median high school teacher's salary

X_6 = number of course units offered

X_7 = building value per pupil in ADA

X_8 = number of pupils in ADA

X_9 = bonded indebtedness per pupil in ADA

X_{10} = number of pupils in ADA/number of teachers

X_{11} = average composite ITED score in 10th grade (class of 1961)

To clarify the nature of the Iowa data concerning variables Y_1, Y_2, and X_2 through X_{11}, we present the following tabulation:

Variable	Units	(1) Mean (N = 378)	(2) Standard deviation (N = 378)	Coefficient of variation [(2)÷(1)]100
Y_1	points	19.513	1.626	--
Y_2	points	4.279	1.023	--
X_2	dollars	407.335	60.688	14.90
X_3	college hours	28.296	6.647	23.49
X_4	assignments	2.215	0.676	30.52
X_5	dollars	5,252.277	608.967	11.59
X_6	course units	33.351	10.184	30.54
X_7	dollars	1,139.092	448.264	39.35
X_8	pupils	286.687	448.989	156.61
X_9	thousands of dollars	157.082	100.965	64.28
X_{10}	pupils	20.081	12.732	63.40
X_{11}	points	15.234	1.397	--

In addition, we have constructed a set of "dummy" (i.e., zero-one) variables, six of which may be classified as "area variables," and four of which represent variation due to differences in the population of the various districts. They are defined as follows:

X_{13} = 1--Mason City, Calmar and Dubuque areas[1]

0--all other areas

X_{14} = 1--Fort Dodge, Estherville, Sheldon and Sioux City areas

0--all other areas

X_{15} = 1--Ottumwa, Burlington areas

0--all other areas

X_{16} = 1--Creston, Council Bluffs area

0--all other areas

X_{17} = 1--Cedar Rapids, Bettendorf areas

0--all other areas

X_{22} = 1--Ankeny area

0--all other areas

X_{18} = 1--districts with population under 2,500

0--all others

X_{19} = 1--districts with population between 2,500 and 5,000

0--all others

---

[1]See areas in Figure 3-1.

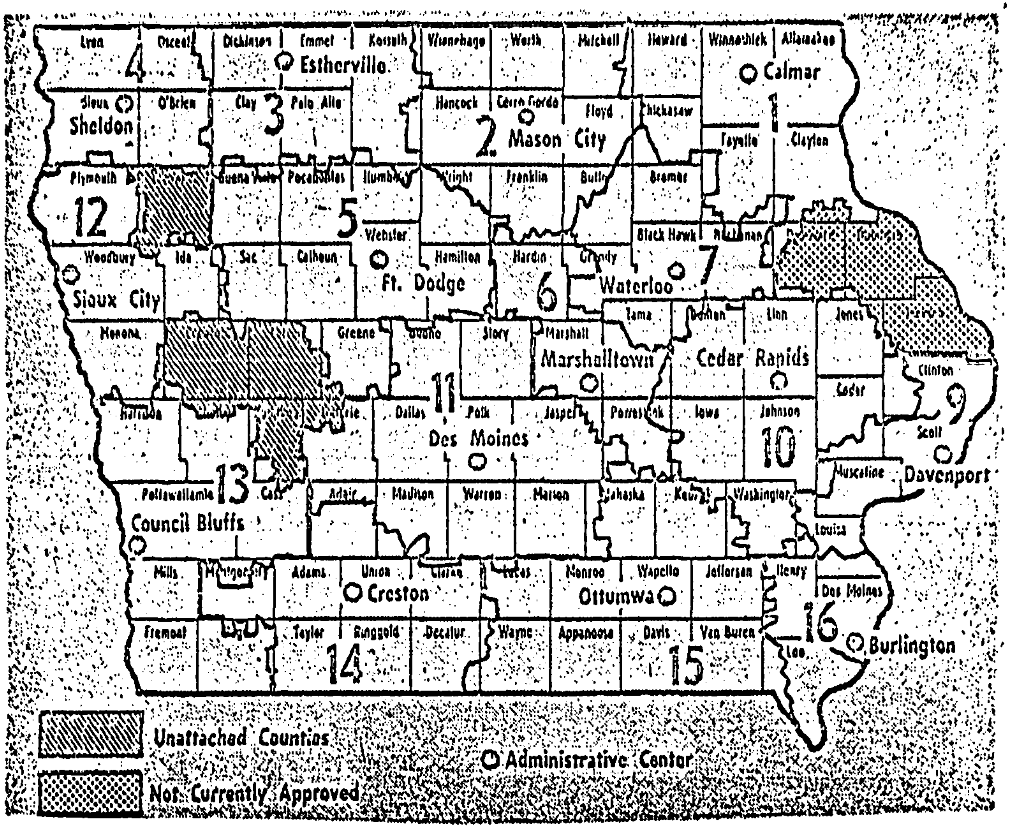

Figure 3-1. Sixteen Iowa school areas (map adapted from the Des Moines Register, September 1, 1967)

X_{20} = 1--districts with population between 5,000 and 10,000

0--all others

X_{21} = 1--districts with population between 10,000 and 50,000

0--all others

Table 3-1 presents the results of multiple regression analyses which we have carried out on models of the general form 3-5. We used, first, the variable Y_2 as the proxy for school quality, inasmuch as we are interested in the contribution of the high school to the development of the student rather than the contribution of the whole educational process from kindergarten through high school. Before any conclusions are attempted, we must note a number of features and characteristics surrounding the ITED.

1. These tests are administered by the school itself, though they are sent to the University of Iowa at Iowa City for processing and grading. While minor variations in the time limit may not be of much importance, it is quite obvious that some difference in the result between any two schools may be due to differences in the administration of the test.

2. While the tests may discriminate well for classes in which students do not reach the "ceiling" of the test, this is not the case for superior students. Suppose that in one school half of the 12th graders are very bright pupils; they may easily reach the ceiling of the test. In fact, they could have made just as high a score on the 10th grade test,

Table 3-1. Multiple regression equations utilizing 378 Iowa high school districts to determine factors influencing school quality (Y_2)[a]

Equation	Intercept	X_3	X_4	X_5	X_6	X_7
I	4.50 (1.33)	- .0187 (.0102)	- .2734 (.1316)	.000178 (.000109)	.0035 (.0105)	- .0000041 (.0001251)
II	4.19 (.71)	- .0178 (.0100)	- .2768 (.1152)	.000194 (.000107)	.0047 (.0099)	.0000054 (.0001224)
III	4.36 (.15)					
IV	4.71 (.41)					
V[b]	.52 (.21)	- .1733 (.0648)	- .1189 (.0815)	.0492 (.0229)	.0136 (.0496)	.0194 (.0339)
VI[b]	.55 (.16)	- .1495 (.0643)	- .1236 (.0764)	.0470 (.0229)	.0142 (.0492)	.0232 (.0329)
VII[b]	.68 (.12)	- .1318 (.0627)	- .1864 (.0484)	.0500 (.0209)		

[a]Numbers in parentheses indicate standard errors of coefficients.

[b]Variables are transformed into logarithms.

Table 3-1. (Continued)

Equation	X_8	X_9	X_{10}	X_{13}	X_{14}	X_{15}
I	.0000014 (.0003885)	.00063 (.00055)	.00299 (.00448)	.0752 (.2092)	-.104 (.185)	-.273 (.250)
II	-.000080 (.000194)	-.000205 (.000527)	.0034 (.0044)			
III				.1335 (.2091)	-.0998 (.1852)	-.164 (.243)
IV						
V[b]	.0602 (.0470)	-.0072 (.0101)	-.0218 (.0380)	.0082 (.0223)	-.0059 (.0198)	-.0334 (.0264)
VI[b]	.0313 (.0336)	.0017 (.0097)	-.0126 (.0375)			
VII[b]			-.0002 (.0352)			

Table 3-1. (Continued)

Equation	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}	X_{21}
I	- .466 (.225)	- .049 (.204)	- .03 (1.11)	.108 (1.094)	.105 (1.062)	- .181 (.953)
II						
III	- .371 (.219)	- .038 (.203)				
IV			- .614 (.421)	- .342 (.424)	- .198 (.439)	- .322 (.451)
V[b]	- .0616 (.0242)	- .0076 (.0216)	.0373 (.0707)	- .0466 (.0644)	.0445 (.0593)	.0066 (.0553)
VI[b]						
VII[b]						

Table 3-1. (Continued)

Equation	X_{22}	Mean Y_2	R^2	$\bar{R}^2$	F	Standard error of estimate
I	-.183 (.215)	4.279	.074	.030	1.612	1.009
II		4.279	.047	.028	2.279	1.010
III	-.113 (.214)	4.279	.017	.004	1.094	1.023
IV		4.279	.027	.019	2.597	1.015
V[b]	-.0200 (.0229)	.6199	.100	.057	2.216	0.1072
VI[b]		.6199	.062	.045	3.088	0.1079
VII[b]		.6199	.058	.050	5.752	0.1076

Table 3-1. (Continued)

Equation	Intercept	X_3	X_4	X_5	X_6	X_7
VIII[c]	4.30	-.0192	-.2372	.00020		
	(.76)	(.0109)	(.1186)	(.00011)		
IX[c]	4.19	-.0205	-.2213	.00019	-.0017	.000028
	(.86)	(.0112)	(.1555)	(.00012)	(.0174)	(.000139)
X[d]	4.35	-.0174	-.4545	.00022		
	(1.22)	(.0215)	(.2682)	(.00017)		
XI[d]	4.38	-.0163	-.3865	.00024	.0040	.00016
	(1.31)	(.0223)	(.2966)	(.00019)	(.0107)	(.00023)

[c]The equation is based on 290 observations of Set 1 (see explanation in text).

[d]The equation is based on 87 observations of Set 2 (see explanation in text).

Table 3-1. (Continued)

Equation	X_8	X_9	X_{10}	X_{13}	X_{14}	X_{15}
VIII[c]		.000006 (.000584)				
IX[c]	.0007 (.0013)	.000003 (.000598)	.0023 (.0046)			
X[d]		.0005 (.0010)				
XI[d]	- .00011 (.00018)	.000018 (.001178)	- .0249 (.0266)			

Table 3-1. (Continued)

Equation	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}	X_{21}
VIII[c]						
IX[c]						
X[d]						
XI[d]						

Table 3-1. (Continued)

Equation	X_{22}	Mean Y_2	R^2	$\bar{R}^2$	F	Standard error of estimate
VIII[c]		4.235	.031	.021	2.320	1.032
IX[c]		4.235	.034	.010	1.237	1.038
X[d]		4.486	.071	.038	1.584	.755
XI[d]		4.486	.096	.016	1.044	.764

resulting in no net change in quality for that group! It is therefore to be expected that in schools where the percentage of bright and well educated pupils exceeds the average for the state of Iowa, the variable Y_2 may not be appropriate.

3. Although academic achievement and intelligence are supposed to be, on the average, highly correlated, there may be cases in which the ITED will reflect the intelligence of the pupils, rather than the quality of the academic program. In other words, it may be necessary to make some corrections for differences in native ability, though, in general, such corrections in school averages do not seem to be of much importance.

4. It must be realized that the ITED cover predominantly the "three R's" and other common areas such as social sciences and basic sciences. These tests, then, fail by their very nature to cover the full scope of the educational program in the school. It is quite likely that schools which are equally good insofar as the subject matters tested in the ITED are concerned differ greatly in the quality, as well as the quantity, of other important subjects. In this sense, the use of Y_1 or Y_2 as a quality index falls far short of the mark. This point will apply in particular for observed quality differences between the large and small high schools. That is, while the latter may well excel in the subjects tested by the ITED, they are more likely (than the former) to lack in the presentation of diverse subject

matters in quantity and quality.

Furthermore, the models utilized below include only the factors for which quantitative data were available. There may be some important factors which should have been included but which were excluded for lack of information. For instance, we do not have sufficient information on the experience of the "average teacher" for each school, and hence could not include an experience variable in the model. Similarly, some information on the socio-economic composition of the population of the district, as well as rates of employment growth and the like, could be quite instructive. It is not surprising, then, that our empirical models do not show as good explanatory power as we would have liked them to demonstrate.

Several sub-models of type 3-5 were tried, some of which are reported in Table 3-1. Equation I of this table is an additive multiple regression equation, in which we have included, in addition to the factor inputs, also the "dummy" variables. The results are somewhat disappointing; but in the light of our previous analysis, it is not surprising. Moreover, we do find some of the factors to contribute significantly (in the statistical sense) to the overall index of quality. Specifically, variables X_3, X_4 and X_5 show a measure of statistical significance. That is to say, the analyses suggest that an increase in the median salary level of Iowa high schools, on the average, will tend to raise the index of quality and that a decrease in the number of assignments per teacher will also result in improved quality; they imply, somewhat surprisingly, that an increase in the average number of college hours per teaching assignment tend, other things equal, to <u>lower</u> the overall level of quality as measured by ITED.

The purpose of Equations II, III and IV of Table 3-1 was to assess the importance of the several groups of variables involved in Equation I. It is immediately apparent that the area variables are of no statistical importance in explaining the variation in Y_2. Equation IV suggests that population of school district may have a slight association with school quality. However, if both sets of the dummy variables are dropped from the model altogether, explanatory power of our model is not significantly affected. To see this, one must compute the value of the corrected R^2 ($\overline{R}^2$ in Table 3-1--the coefficient of determination corrected for the degrees of freedom). A convenient formula has been obtained by R. J. Wheery (42). Let $\overline{R}$ be the estimated correlation obtaining in the universe, R the observed multiple correlation coefficient, M the number of independent variables, and N the number of observations. Then the corrected R^2 is given by

$$(3\text{-}3) \qquad \overline{R}^2 = \frac{(N - 1)R^2 - (M - 1)}{N - M}$$

The appropriate values for $\overline{R}^2$ have been computed for selected equations, and are reported in some of the tables. A casual observation of the results of the use of this formula is striking: the "real" reduction in R^2 due to the omission of some ten dummy variables is only 0.002--not .027 as we would have thought had the $\overline{R}^2$'s not been computed.

An attempt to improve the "fit" of the model was made in Equations V, VI and VII. For these equations, the variables--all except the dummy ones--were transformed into logarithms. In essence, we assumed that instead of an additive model, a multiplicative one may have been more appropriate. On the whole, the logarithmic model performs better, although the difference is not very great. Whereas the full additive model "explains" about 3 per cent of the total variation in quality (after corrections) the corresponding logarithmic model "explains" as much as 5.7 per cent. But we still explain only a very small portion of the variation in Y_2. A comparison of the results of Equations VI and VII suggests, in addition to the conclusion about the dummy variables obtained for the additive model, that many of the factor inputs (taken as a group) contribute little or nothing to the explanatory power of the model. We are left, once more, with variables 3, 4 and 5 as the variables of possible explanatory value.

As we have mentioned above, it appears, on intuitive grounds, that the quality indices for the larger school districts may well be different from those of the smaller ones. In an attempt to gauge the statistical nature of such alledged differences, we divided the data into two sets. Set 1 contains all districts whose total populations in 1960 were less than 5,000. Set 2, in turn, contains the districts whose total populations were 5,000 or more in 1960.[1] Some of the results of this investigation are shown in Table 3-1. The differences between the results for the two size groups are not significant.

So far we have used the difference between the test scores that were obtained in 12th grade and those which were obtained in the 10th grade as the measure of quality. There may be some justification for the use of the 12th grade score, Y_1, as the index of quality, particularly because of the "ceiling" problem which we had occasion to mention before. In essence, two models seem appropriate; one would simply substitute Y_1 for Y_2, while the second would, in addition, include X_{11} as an independent variable. The rationale for the use of the second model is, perhaps, at the core of the economics of education, namely, that not only factors which use (or sacrifice) physical capital are of importance; the human element as such is also an important factor of production. In that sense, the 10th grade score underlies the basic human element

[1]Data on the districts' populations were obtained from (22). See also Table 3-4.

with which the school must operate, and hence it may represent that important human element. A number of specific investigations were made with each of the above models, many of which are reported in Tables 3-2 and 3-3 below.

Some increase in explanatory power is achieved by the use of the first model. Further, the coefficient of X_3 comes out to be positive--as we would expect it to be _a priori_. In addition, we have some evidence (Equations II and III of Table 3-2) that the dummy variables, taken separately, do explain a significant, though very small, portion of the variation in Y_1. Another interesting result of the model can be seen when Equations IV and V of Table 3-2 are compared. In that comparison, the model "explains" much better for Set 2 (see above) than for Set 1.

Turning our attention to the second model (where X_{11} is included as an independent variable), one observation is that R^2 increases a great deal (compared to all previous models). The reason for this is the very large simple correlation between X_{11} and Y_1 (which, in the original data, equals to 0.7808). Once again, many versions of the model were attempted, and the results, except for the higher R^2, as well as the addition of X_{11} as a "key" factor, are not much different (that is, only X_3, X_4 and X_5 come out to be significant; the dummy variables do not add much to the total explanatory power of the model; and the coefficient of X_3 still turns out

Table 3-2. Factors affecting school quality, Y_1, for 378 Iowa high schools (1961-62)[a]

Equation	Mean Y_1	Intercept	X_3	X_4	X_5	X_6
I	19.51	16.94	.0225	- .1861	.000489	- .01357
		(2.04)	(.0156)	(.2014)	(.000168)	(.01620)
II	19.51	19.77				
		(.24)				
III	19.51	19.88				
		(.65)				
IV[b]	19.34	18.52	.01414	- .2616	.000465	- .0219
		(1.42)	(.01750)	(.2277)	(.000189)	(.0235)
V[c]	20.10	14.56	.0336	- .0610	.000570	.0047
		(2.41)	(.0355)	(.5069)	(.000329)	(.0160)

[a]Numbers in parentheses indicate standard errors of coefficients.

[b]Based upon 290 districts whose populations, as of 1960, were below 5,000.

[c]Based upon 87 districts whose populations, as of 1960, were 5,000 or more.

Table 3-2. (Continued)

Equation	X_7	X_8	X_9	X_{10}	X_{13}	X_{14}
I	-.000033	.000282	-.001178	-.00571	-.4693	.1533
	(.000191)	(.000594)	(.000847)	(.00686)	(.3202)	(.2832)
II					-.443	.1159
					(.329)	(.2918)
III						
IV[b]			-.00135	-.00418	-.5844	-.0267
			(.00095)	(.00730)	(.3793)	(.3331)
V[c]			.00139	-.0128	-.0389	.9986
			(.00200)	(.0446)	(.5397)	(.5159)

Table 3-2. (Continued)

Equation	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}
I	-.9147	-.6239	-.4017	.6824	.762	1.008
	(.3834)	(.3445)	(.3125)	(1.7015)	(1.675)	(1.626)
II	-.7402	-.6394	-.4050			
	(.3833)	(.3455)	(.3212)			
III				-.6366	-.4260	.1554
				(.6643)	(.6683)	(.6915)
IV[b]	-.5505	.7110	-.6235		.0772	
	(.4717)	(.4029)	(.3775)		(.2241)	
V[c]	-1.291	-.0571	-.0226			1.0731
	(.5922)	(.6069)	(.5087)			(.7759)

Table 3-2. (Continued)

Equation	X_{21}	X_{22}	R^2	$\bar{R}^2$	F	Standard error of estimate
I	.9641 (1.4595)	- .385 (.329)	.1413	.1007	3.283	1.544
II		- .2778 (.3381)	.0342	.0212	2.192	1.611
III	.3635 (.7113)		.0439	.0362	4.286	1.598
IV[b]		- .4884 (.3978)	.0848	.0451	1.967	1.613
V[c]	1.1376 (.7172)	- .1762 (.5199)	.3700	.2578	3.021	1.195

Table 3-3. Factors influencing school quality, Y_1, for 378 Iowa high schools (1961-62)[a]

Equation	Mean Y_1	Intercept	X_3	X_4	X_5	X_6
I	19.51	5.822	-.0141	-.2575	.000214	.0016
		(1.412)	(.0102)	(.1303)	(.000109)	(.0105)
II	19.51	5.669	-.0134	-.2673	.000241	
		(.810)	(.0097)	(.1019)	(.000100)	
III	19.51	5.646	-.0127	-.2692	.000240	.00204
		(.862)	(.0101)	(.1142)	(.000107)	(.00987)
IV[b]	1.289	.428	-.03049	-.02053	.01005	.00373
		(.052)	(.01330)	(.01654)	(.00464)	(.01007)
V[b]	1.289	.431	-.0256	-.0235	.00970	.00325
		(.044)	(.0132)	(.0154)	(.00464)	(.01000)
VI[b]	1.289	.458	-.0222	-.0334	.01045	
		(.039)	(.0129)	(.0098)	(.00425)	

[a]Numbers in parentheses indicate standard errors of coefficients.

[b]Variables (excluding X_{12} - X_{22}) are transformed into logarithms.

Table 3-3. (Continued)

Equation	X_7	X_8	X_9	X_{10}	X_{11}	X_{13}
I	- .0000067 (.0001238)	.000059 (.000384)	- .000692 (.000548)	.00192 (.00445)	.8866 (.0396)	.0229 (.2083)
II				.00249 (.00432)	.8857 (.0380)	
III	.000010 (.000121)	- .000066 (.000193)	- .000245 (.000522)	.00257 (.00438)	.8851 (.0385)	
IV[b]	.00587 (.00688)	.01108 (.00955)	- .00205 (.00206)	- .00416 (.00772)	.7026 (.0297)	.00139 (.00455)
V[b]	.00659 (.00669)	.00503 (.00682)	- .00021 (.00198)	- .00200 (.00761)	.7055 (.0289)	
VI[b]				- .00006 (.00715)	.7072 (.0286)	

Table 3-3. (Continued)

Equation	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}
I	- .0766 (.1834)	- .3442 (.2493)	- .5061 (.2229)	- .0905 (.2026)	.1225 (1.1010)	.2575 (1.0840)
II						
III						
IV[b]	- .00050 (.00402)	- .0086 (.0054)	- .0117 (.0049)	- .00220 (.00440)	.00711 (.01436)	.0091 (.0130)
V[b]						
VI[b]						

Table 3-3. (Continued)

Equation	X_{20}	X_{21}	X_{22}	R^2	$\bar{R}^2$	F	Standard error of estimate
I	.2838 (1.0527)	-.0174 (.9451)	-.2024 (.2134)	.6416	.6237	33.74	.999
II				.6299	.6259	126.63	.996
III				.6302	.6221	69.69	1.001
IV[b]	.0091 (.0120)	.0022 (.0112)	-.00333 (.00465)	.6612	.6441	36.67	.021
V[b]				.6476	.6399	74.95	.021
VI[b]				.6459	.6420	135.36	.021

Table 3-3. (Continued)

Equation	Mean Y_1	Intercept	X_3	X_4	X_5	X_6
VII[c]	19.34	5.737 (1.040)	-.0164 (.0112)	-.2137 (.1540)	.000232 (.000122)	-.0065 (.0173)
VIII[d]	20.10	4.822 (1.591)	-.0124 (.0229)	-.3144 (.2998)	.000276 (.000203)	.0035 (.0109)
IX[b,c]	1.28	.433 (.053)	-.0277 (.0146)	-.0142 (.0199)	.0112 (.0055)	-.0049 (.0174)
X[b,d]	1.30	.085 (.193)	-.0090 (.0375)	-.0293 (.0250)	.1029 (.0569)	.00745 (.00968)

[c]Based upon 290 districts whose populations, as of 1960, were below 5,000.

[d]Based upon 87 districts whose populations, as of 1960, were 5,000 or more.

Table 3-3. (Continued)

Equation	X_7	X_8	X_9	X_{10}	X_{11}	X_{13}
VII[c]	.000032 (.000138)	.00085 (.00135)	-.000102 (.000593)	.00162 (.00466)	.8863 (.0432)	
VIII[d]	.000169 (.000237)	-.000115 (.000185)	.00018 (.00120)	-.0291 (.0269)	.9484 (.0800)	
IX[b,c]	.0046 (.0084)	.0164 (.0103)	-.00036 (.00222)	-.00216 (.00844)	.6934 (.0335)	
X[b,d]	.0136 (.0104)	-.0130 (.0097)	-.00204 (.00532)	-.0251 (.0257)	.7267 (.0605)	

Table 3-3. (Continued)

Equation	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}
VII[c]						
VIII[d]						
IX[b,c]						
X[b,d]						

Table 3-3. (Continued)

Equation	X_{20}	X_{21}	X_{22}	R^2	$\bar{R}^2$	F	Standard error of estimate
VII[c]				.6230	.6122	51.41	1.028
VIII[d]				.7196	.6908	21.95	.771
IX[b,c]				.6222	.6114	51.25	.023
X[b,d]				.7333	.7059	23.53	.016

Table 3-4. Means and standard deviations of selected variables and models[a]

Model	Y_2	Y_1	X_3	X_4	X_5	X_{11}
Additive, 378 observations	4.279 (1.023)	19.51 (1.62)	28.29 (6.64)	2.215 (.676)	5,252.77 (608.96)	15.23 (1.397)
Logarithmic, 377 observations	.6199 (.1103)	1.2890 (.0364)	1.4390 (.1087)	.3233 (.1430)	3.7013 (.2732)	1.1808 (.04030)
Additive, 149 observations	4.330 (.996)	19.52 (1.42)	28.08 (7.10)	2.30 (.733)	5,218.42 (630.60)	15.19 (1.23)
Additive, 290 observations (Set 1)	4.235 (1.041)	19.34 (1.64)	26.50 (5.93)	2.454 (.551)	5,105.44 (548.69)	15.10 (1.43)
Additive, 88 observations (Set 2)	4.486 (.765)	20.10 (1.37)	34.29 (5.31)	1.415 (.374)	5,745.03 (543.59)	15.62 (1.15)
Additive, 166 observations[b]	4,142 (1.109)	19.25 (1.86)	25.70 (6.52)	2.654 (.537)	5,003.94 (649.06)	15.11 (1.58)

[a]Figures in parentheses are standard deviations of variables.

[b]Data are for all districts whose 1960 populations were under 2,500.

Table 3-4. (Continued)

Model	Y_2	Y_1	X_3	X_4	X_5	X_{11}
Additive, 124 observations[c]	4.361 (.932)	19.46 (1.29)	27.57 (4.84)	2.187 (.448)	5,241.33 (331.24)	15.10 (1.22)
Additive, 49 observations[d]	4.518 (.803)	20.03 (1.56)	32.44 (4.44)	1.583 (.385)	5,528.38 (449.40)	15.52 (1.23)
Additive, 38 observations[e]	4.444 (.722)	20.18 (1.10)	36.67 (5.43)	1.198 (.217)	6,024.39 (531.52)	15.77 (1.04)

[c]Data are for all districts whose 1960 populations were between 2,000 and 5,000.

[d]Data are for all districts whose 1960 populations were between 5,000 and 10,000.

[e]Data are for all districts whose 1960 populations were over 10,000.

to be negative) from those achieved by models utilizing Y_2 as the index of quality (see Table 3-3). Finally, a logarithmic version of the model was also tried, and a comparison of such equations as II and VI of Table 3-3 suggests that the multiplicative model provides a (very) slightly better fit than the additive model.[1]

A Sampling Experiment

Prior to the analysis of the full data set (of 378 school districts) we chose a somewhat "representative" sample of five areas (Mason City area, Fort Dodge area, Ottumwa area, Creston area and the Cedar Rapids area) in which 149 school districts were observed.[2] It was felt that these areas encompass a great deal of the different economic regions in Iowa, and therefore the results should not be much different than if the whole state was our geographical base.

The results of our investigation are presented in Table 3-5. It must be noted that a direct comparison with our previous models cannot be made, insofar as the dummy variables are concerned, because variables X_{14} through X_{17} were defined differently (i.e., X_{14} = 1 for Fort Dodge area, 0 otherwise;

[1]In addition, a multiplicative model is useful in that the coefficients which we obtain are elasticities rather than slopes. That is, if the coefficient of variable X_i is a_i, the implication is that if X_i is changed by 1 per cent, the dependent variable, Y, will change by a_i per cent.

[2]These areas can be observed in Figure 3-1.

Table 3-5. Multiple regression equations determining factors influencing school quality: 149 districts[a]

Equation	Intercept	X_3	X_4	X_5	X_6	X_7	X_8
I[b]	6.12 (3.46)	-.0257 (.0143)	-.2020 (.1948)	.000188 (.000156)	.0036 (.0173)	.000042 (.000213)	-.000355 (.000925)
II[b]	3.80 (1.18)	-.0250 (.0142)	-.2933 (.1707)	.000271 (.000152)	.0086 (.0145)		-.000245 (.000324)
III[b]	4.12 (1.10)	-.0267 (.0139)	-.2990 (.1695)	.000255 (.000149)	.0017 (.0110)		
IV[c]	9.50 (3.55)	-.0179 (.0141)	-.2413 (.1896)	.000199 (.000151)	-.0031 (.0169)	.000038 (.000207)	-.000267 (.000898)
V[c]	19.51 (1.62)	.01318 (.01941)	-.5655 (.2328)	.000402 (.000207)	-.0383 (.0198)	.000091 (.000277)	.000451 (.000442)
VI[c]	7.07 (1.47)	-.0180 (.0139)	-.3491 (.1661)	.000292 (.000145)	-.0038 (.0109)		

[a]Numbers in parentheses indicate standard error of coefficients.

[b]Dependent variable is Y_2, whose mean $\overline{Y}_2 = 4.33$.

[c]Dependent variable is Y_1, whose mean $\overline{Y}_1 = 19.52$.

Table 3-5. (Continued)

Equation	X_9	X_{10}	X_{11}	X_{14}	X_{15}	X_{16}	X_{17}
I[b]		.00517 (.00568)		-.5178 (.2577)	-.8282 (.3064)	-.7339 (.3145)	-.5764 (.2537)
II[b]	.000973 (.000873)	.00565 (.00561)					
III[b]	.000945 (.000806)	.00561 (.00558)					
IV[c]	-.000202 (.000935)	.00184 (.00562)	.7907 (.0698)	-.4212 (.2523)	-.9097 (.2988)	-.7324 (.3054)	-.5595 (.2465)
V[c]	.00029 (.00119)	-.00788 (.00766)					
VI[c]	.000854 (.000786)	.00294 (.00552)	.8011 (.0681)				

Table 3-5. (Continued)

Equation	X_{18}	X_{19}	X_{20}	X_{21}	F	R^2
I[b]	- 1.455	- 1.181	- .958	- 1.027	1.61	.163
	(3.261)	(3.235)	(3.177)	(2.728)		
II[b]					1.68	.087
III[b]					2.17	.084
IV[c]	- 1.562	- 1.298	- 1.039	- 1.007	12.46	.618
	(3.167)	(3.142)	(3.086)	(2.649)		
V[c]					3.64	.172
VI[c]					27.66	.578

X_{15} = 1 for Ottumwa area; X_{16} = 1 for Creston area, and X_{17} = 1 for the Cedar Rapids area, and 0 otherwise). On the other hand, one may compare, say, Equations II of Table 3-5 and II of Table 3-1. And while the size of the coefficients, as well as the R^2's are not precisely the same, it may be said that much of the information that Table 3-5 provides us would lead us to the same conclusions that were obtained by analyzing the full data set.

Some Concluding Remarks

An attempt has been made to estimate the quantitative effects of some of the factors that may affect school quality Y_1 or Y_2 in Iowa. More refined methods can surely be devised, and much is constantly done by educators to improve the usefulness of such devices as the ITED. Although our models have not exhibited much explanatory power, some things have been learned in the process. A few factors of the educational input system proved to be statistically significant in their relation to changes in the level of quality, however we chose to define it. Note, however, that the standard errors of the coefficients of X_3, X_4 and X_5 are quite large, so that we cannot claim very much for the results on statistical grounds.

We have not yet had our last say on the use and construction of operational quality models. But before an "alternative approach" is taken in Chapter 5, we wish, first, to turn our attention to an important and interesting application of the models which we have developed and tested thus far. This will be done in Chapter 4.

We must also recognize that an aggregation problem exists, so that implications from any quality model must be drawn with extra caution (see the Appendix for a detailed discussion).

Table 3-6. Simple correlation coefficients of the variables discussed in the text (for the 378 Iowa districts, 1961-62).

	Y_2	X_2	X_3	X_4	X_5	X_6
Y_2	1.0000					
X_2	-0.0266	1.0000				
X_3	0.0358	-0.0827	1.0000			
X_4	-0.1694	0.2998	-0.5507	1.0000		
X_5	0.1592	-0.0668	0.4561	-0.4786	1.0000	
X_6	0.1275	-0.2148	0.4917	-0.6309	0.5351	1.0000
X_7	0.0251	0.3962	0.1382	0.0049	0.1507	-0.0348
X_8	0.0878	-0.1988	0.4310	-0.5038	0.4292	0.8007
X_9	0.0070	-0.0180	0.1051	-0.0430	0.0248	0.0256
X_{10}	-0.0352	-0.0618	-0.0749	0.3339	-0.0974	-0.1034
X_{11}	-0.1047	0.0060	0.2311	-0.1390	0.2073	0.0643
Y_1	0.5200	-0.0087	0.2191	-0.2199	0.2742	0.1317

Table 3-6. (Continued)

	X_7	X_8	X_9	X_{10}	X_{11}	Y_1
Y_2						
X_2						
X_3						
X_4						
X_5						
X_6						
X_7	1.0000					
X_8	0.0167	1.0000				
X_9	0.1801	0.0163	1.0000			
X_{10}	-0.0313	-0.0506	-0.0157	1.0000		
X_{11}	0.0593	0.0709	-0.0213	-0.0777	1.0000	
Y_1	0.0670	0.1135	-0.0128	-0.0879	0.7944	1.0000

CHAPTER FOUR. ECONOMIES OF SCALE IN HIGH SCHOOL OPERATIONS

Although our analysis of the quality models based upon the results of the ITED (Iowa Tests of Educational Development) scores for Iowa high schools did not fulfill our hopes concerning the use of such models for educational policy-making, the same variables yield an interesting by-product. This by-product should be of interest to school administrators who may have some control over school size, if they desire to arrive at an "optimal" decision from an ecomic standpoint.

A case in a point may be the question of consolidation. While there exist many problems in any such endeavor, the subject of economies of scale is often considered to be of overriding importance. Thus, casual observation of the Iowa data for 1962-63 school years reveals that school size and total expenditures per pupil are definitely related--the smaller the school (in terms of the average daily attendance of high school students) the higher the level of expenditures per pupil (in ADA), on the average, are expected to be. However, if consolidation implies loss in the quality of instruction, the expected lower per pupil costs are quite misleading. On the other hand, if consolidation reinforces the quality of instruction, the mere casual observation of reduced per pupil costs does not tell the extent of the "true" benefits from consolidation.

It is, therefore, necessary to make "corrections" for quality differences in the final effect of school size upon costs per pupil. Second, given that such corrections are made, it may be possible to find an "optimum" size of a high school (for a given area and time period, of course), although, as we shall see, such an optimum is not so easy to construct. Also, in the course of the chapter, some tentative results will be presented as to the existence of economies of scale in Iowa high school operations and on the "optimum" size of a high school for Iowa (for the period 1962-63).

Corrections for Quality Differences

The first to attempt a "corrected economies of scale" model was John Riew (33) in an article in which examined possible economies of scale in Wisconsin high school operations. His model, basically, is

$$(4\text{-}1) \qquad E = f(Q, ADA)$$

where E denotes expenditures per pupil, Q is an index of school quality, and ADA is the average daily attendance record for the high school. Specifically, Riew's model contains the following multiple regression (single equation) model:

$$(4\text{-}2) \qquad X_2 = a + bX_8 + cX_8^{\,2} + dX_4 + eX_5 + fX_6 + gZ_1 + hZ_2$$

where X_2, X_4, X_5 and X_6 are as defined in Chapter 3 (except for minor variations as to the exact definitions of each variable), and

Z_1 = change in enrollment between 1957 and 1960

Z_2 = per cent of classrooms built after 1950.

While model 4-2 does correct for quality differences, "quality" is defined by a combination of the variables X_4, X_5 and X_6. These variables were chosen not by recourse to some objective criterion, such as the ITED, but merely by the use of accepted beliefs.

A particularly interesting version of 4-1 would be

$$(4\text{-}3) \qquad X_2 = a + bX_8 + cX_8^2 + dY_2$$

where Y_2 is the difference between the ITED composite score achieved at the 12th grade level and that of the 10th grade level. In addition, other versions of 4-1 may be formulated, some of which are reported in Table 4-1. In each case X_2, operating expenditures per pupil, is the dependent variable and X_8, the number of pupils in average daily attendance, is the independent variable of primary importance.

It is apparent that, whichever specific model one may wish to choose, we can conclude with a high degree of (statistical) significance that economies of scale do exist in Iowa high schools, even after differences in quality are taken into account. Further, Equations IV and V of Table 4-1, in which the coefficient of X_8^2 is significantly positive,

Table 4-1. Factors influencing expenditures per pupil for 377 Iowa high school districts (1962-63)[a]

Equation	$\bar{X}_2$[c]	Intercept	X_8 (ADA)	X_8^2	X_3	X_4	X_5
I[b]	2.283	2.78	-.099				
		(.02)	(.008)				
II[b]	2.283	2.47	-.093		.0697	.0278	-.021
		(.07)	(.015)		(.0298)	(.0354)	(.010)
III	287.30	437.00	-.147	.000049			
		(5.04)	(.020)	(.000008)			
IV	287.30	433.70	-.147	.000049			
		(13.19)	(.020)	(.000008)			
V	287.30	263.45	-.177	.0000537	1.14	20.20	.00404
		(34.82)	(.031)	(.0000099)	(.49)	(6.25)	(.00525)

[a]Numbers in parentheses indicate standard errors of coefficients.

[b]Variables are transformed into the logarithms.

[c]$\bar{X}_2$ = mean of expenditures per pupil.

Table 4-1. (Continued)

Equation	X_6	X_7	X_9	X_{10}	Y_2	F	R^2
I[b]					.0064 (.0262)	42.12	.190
II[b]	.0702 (.0228)	.1068 (.0152)	- .020 (.004)	- .0636 (.0174)		24.25	.345
III						27.19	.1269
IV					.79 (2.94)	18.11	.1271
V	1.357 (.512)	.053 (.006)	- .061 (.026)	- .639 (.218)		20.85	.338

suggest that diminishing marginal returns are likely to occur beyond a certain point.

An Optimal School Size

In his paper, Riew argued that a model such as Equation 4-2 above can serve to estimate the optimal size of a high school (or, in our case, perhaps a high school district size). His argument is thus: Form

$$(4\text{-}4) \qquad X_2 = a + bX_8 + cX_8^{\,2} + b_qQ$$

where b_qQ is a composite index of quality (and other variables). Now take

$$(4\text{-}5) \qquad \partial X_2/\partial X_8 = b + 2cX_8$$

and set the result in 4-5 equal to 0. Then we get:

$$(4\text{-}6) \qquad X_8^* = -b/2c$$

where X_8^* refers to the optimal ADA for a high school.

The results which Riew obtained for Wisconsin (optimal ADA = 1,675) are not without their pitfalls. In the first place, just because a quadratic component seems to be significant is not necessarily an indication that diminishing <u>total</u> returns ever set in. It may well be that optimal school size is anywhere between 1,675 and infinity. That is, using Equation 4-7, $\partial X_2/\partial X_8 = -cX_8^{-2}$ and $\partial X_2/\partial X_8 = 0$ only when X_8^* approaches infinity. To illustrate the point, one may fit the same data for a rectangular hyperbola, i.e., we form:

$$(4\text{-}7) \qquad X_2 = a + b_q Q + cX_8^{-1}$$

Equations I and II of Table 4-2 present two variants of model 4-7. It appears that, for the case of Iowa, a model such as 4-7 better fits the data than its counterpart in 4-4. Hence, it cannot be argued forcefully that a true optimum school size can actually so easily be determined. We obviously need more information.

In addition, one should obtain confidence limits for X_8^*. A simple procedure for obtaining such limits can be illustrated as follows.[1] Let

$$R = w_2 / w_1$$

where $w_2 = -b$, $w_1 = 2c$, and $R = X_8^*$. Now the statement $R = w_2 / w_1$ is identical to the statement $w_2 - Rw_1 = 0$. In this procedure, "the known method of setting confidence limits to the difference $w_2 - Rw_1$ is employed to determine a confidence interval for R." Since[2]

$$\begin{aligned} Var(w_2) &= c_{22}s^2 \\ Var(w_1) &= c_{11}s^2 \\ Cov(w_1, w_2) &= c_{12}s^2 \end{aligned}$$

---

[1]This procedure is presented by Fuller (16, pp. 82-86).

[2]The c_{ij} are elements of a 2 x 2 matrix C, where Cs^2 is the estimated variance-covariance matrix of w_1 and w_2.

Table 4-2. Multiple regression equations determining factors described in model 4-7 for 377 Iowa high school districts (1962-63)[a]

Equation	Intercept	X_8^{-1}	X_3	X_4	X_5	X_6
I	362.11 (5.15)	6,831.97 (667.85)				
II	253.62 (32.51)	7,240.41 (957.44)	.703 (.473)	7.45 (6.54)	.0029 (.0050)	.536 (.350)

[a]Numbers in parentheses indicate standard errors of coefficients. Also, the value of $\overline{X}_2$ = 287.30.

Table 4-2. (Continued)

Equation	X_7	X_9	X_{10}	F	R^2
I				104.64	.218
II	.0476 (.0058)	- .037 (.025)	- .587 (.211)	27.93	.377

Table 4-3. Simple correlation coefficients for the Iowa data (377 observations) and the variables used in the text[a]

	X_2	X_8	X_8^{-1}	X_8^{2}
X_2	1,0000	-0.2004	0.4671	-0.0972
X_8	-0.2004	1.0000	-0.4798	0.9506
X_8^{-1}	0.4671	-0.4798	1.0000	-0.2805
X_8^{2}	-0.0972	0.9506	-0.2805	1.0000
Y_2	-0.0698	0.0917	-0.1542	0.0602
X_3	-0.0855	0.4313	-0.4541	0.2947
X_4	-0.2186	-0.5041	0.7436	-0.3160
X_5	-0.0673	0.4293	-0.4141	0.3119
X_6	-0.2186	0.8008	-0.6188	0.6632
X_7	0.4071	0.0136	0.0736	0.0175
X_9	0.0011	0.0133	-0.0585	0.0213
X_{10}	-0.0678	-0.0495	0.1280	-0.0209

[a]For other correlation coefficients see Table 3-6 of Chapter 3.

"it follows that the α confidence interval of $[\ R = X_8^*\ ]$ is defined by those values of R such that

$$\text{Prob}\left\{\frac{(w_2 - Rw_1)^2}{(c_{22} - 2Rc_{12} + R^2c_{11})s^2} \leq t^2_{1-\alpha}\right\} = \alpha$$

that is, those values of R such that"

$$R^2(w_1^2 - t^2s^2c_{11}) - 2R(w_1w_2 - t^2s^2c_{12}) + w_2^2 - t^2s^2c_{22} \leq 0. \tag{4-8}$$

Indeed, the "optimum" Iowa high school (district) size appears to be about 1,470 (pupils in ADA) if Equations III or IV of Table 4-1 are used, while if Equation V is used, the optimum size increases to about 1,500. Utilizing formula 4-8 above, the lower and upper confidence limits of X_8^* = 1,470 appear to be 1,277 and 1,663 respectively (for α = .05).

Perhaps some insight may be gained, in our case, when we observe the <u>specific</u> relationship between X_2 and X_8 for those districts whose ADA is greater than our "optimum" of 1,470 (pupils in ADA).[1] The figures are reported in Table 4-4.

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[1]Note, however, that no correction is made for differences in quality. Such a correction is called for, in particular, for school no. 5 (in Table 4-4), for which we have the following information: Y_2 = 3.9, X_3 = 36.07, X_4 = 1.04, X_5 = 5,213, X_6 = 48.0, and X_{10} = 31.0. It seems that this school is able to cut per pupil expenditures by offering comparatively low salaries and maintaining a large students-teacher ratio.

Table 4-4. Expenditures per pupil and ADA for districts whose ADA exceeds 1,400 pupils

School no.	Expenditures per pupil	ADA
1	353	1,449
2	392	1,557
3	386	1,571
4	389	1,825
5	287	2,913
6	369	3,308
7	346	3,506
8	444	3,890
9	447	4,115

Conclusions

1. We have presented a method by which a quality index could be used in correcting for quality differences among high schools.

2. Significant economies of scale were found to exist in Iowa high school operations. Also, diminishing marginal returns are found to set in at a certain point. In other words, a larger school is likely to be able to spend a smaller amount of resources per student for the same quality

of education. Other things equal, this seems to imply that consolidation is likely to pay off. However, other things do not necessarily remain equal (transportation costs, for example, are likely to increase), so that the policy implications of our results--for the state of Iowa--are limited until or unless these other costs are appraised simultaneously with money costs to the school districts.

3. An "optimum" scale size was estimated (for Iowa) to be between 1,470 and 1,500 (pupils in average daily attendance).[1] The 5 per cent confidence limits associated with the estimated optimum of 1,470 pupils are 1,277 and 1,663. However, the upper limit of 1,663 reflects our use of a second degree parabola in this particular equation. A rectangular hyperbola, on the other hand, gives an even better fit to our data, and we conclude on this and other grounds that there may be no basis for specifying an upper limit to optimal school size within the range of our Iowa data.

[1]The average daily attendance figures are for the high school alone, not for the school system (including elementary) as a whole.

CHAPTER FIVE. AN ALTERNATIVE APPROACH

In Chapter 3, we have stated the fundamental theme to be pursued insofar as educational policy-making for high schools is concerned. To recapitulate, the educator is likely to be interested in maximizing a certain objective function of the form

$$(5\text{-}1) \qquad f(X_1, X_2, \ldots, X_n) = \sum_j c_j x_j, \quad j=1, 2, \ldots, n$$

where the X_i are the various factors that have influence upon the object to be maximized, namely, school quality. (It may well be, however, that something other than school quality is to be maximized. In some instances, the school board may want to maximize the time available for students to help their parents' farming operations with school quality as a constraint rather than the objective. Furthermore, the definition of "quality" is almost certain to vary between schools, and even within school jurisdictions a consensus as to the meaning of quality is not likely to exist.)

Some of the constraints which must be taken into account in the maximization process have also been described in Chapter 3. Specifically, we stated the following constraints in an illustrative case:

$$
\begin{array}{lr}
a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + a_{14}X_4 + a_{15}X_5 + \ldots + a_{in}X_n & \leq B_o \\
a_{21}X_1 & \leq 4 \\
\quad a_{32}X_2 & \leq 35 \\
\quad\quad a_{43}X_3 & \leq 1{,}000 \\
\quad\quad\quad a_{54}X_4 & \leq 25
\end{array} \tag{5-2}
$$

It has been pointed out earlier that any analysis that purports to "solve" the maximization problem with its attendant constraints, as in 5-1 and 5-2, must provide the necessary objective function weights c_j. Chapter 3 analyzed an attempt to provide these on the basis of their effects on the difference between the composite score on the ITED (Iowa Tests of Educational Development) for the 12th grade and that for the 10th grade. However, the results of that chapter were not conclusive. A further analysis of the data, as well as the theoretical implications of the quality models of Chapter 3, calls for a fundamental change in approach.

In the first place, Y_2 (the change in the ITED scores) may not seriously be regarded as the <u>only</u> measure of high school quality. Many factors that enhance high school quality will, accordingly, have little or no correlation with Y_2--hence the low explanatory power of the regression models of type 3-2 of Chapter 3. For example, the number of credit-units offered by the school shows the extent of curriculum breadth which the high school is able and willing to offer its students. Hypothetically, the more courses that are available for the students from which they can choose, the greater the expected quality that will be generated from the particular high school program. Yet the correlation coefficient between Y_2 and X_6 (credit-units offered) is only 0.1275. Intuitively, we would expect a considerably higher correlation coefficient than this if school administrators who strove to broaden curricula strove with equal energy to increase the change in ITED scores.

Another example may be the size of the "average high school class." While there is apparently a controversy among educators as to the relevance of this factor insofar as the quality of the educational program of the high school is concerned, it seems that, other things equal, a smaller class (i.e., a smaller pupils-to-teachers ratio) will enable closer contacts between pupils and teachers, and hence will permit the teachers to gain more knowledge as to the progress of each child than would have been the case in a large class.

Moreover, teacher salaries reflect the price of teacher services. But it may be necessary to adjust median teachers' salaries for a number of factors that will be discussed shortly. Therefore, a closer analysis of the factors affecting teachers' salaries is called for.

In sum, we ought to devise a new index of quality, one that may include Y_2, but which, in addition, contains other factors on which there exists some consensus among educators as to their importance in the determination of school quality. Specifically, we may formulate a quality index, Q, which is a weighted average of the various school inputs which, in our view, reflect school quality:

$$(5\text{-}3) \qquad Q = f(Z_1, Z_2, \ldots, Z_6)$$

where the Z's are defined as follows:

Z_1 = an index of teaching aids, supervisory personnel and the design and condition of the plant

Z_2 = class size (the students-teacher ratio)

Z_3 = number of college hours per teaching assignment

Z_4 = assignments per teacher

Z_5 = median high school teachers' salary

Z_6 = number of credit-units offered

The Use of Arrow Diagrams

Our principal hypotheses concerning the ways in which various factors affect high school quality are summarized in an arrow diagram (Figure 5-1). Following March and Simon (29), a (+) or (-) sign is attached to each arrow denoting the nature of the relationship between the two variables that are connected by that arrow. For instance, the distance of the high school from the "functional economic area" central city is hypothesized to exert some negative influence on teachers' salaries, while ADA seems to affect the number of units offered in a (strongly) positive manner.

One may note that a new variable, call it "aspiration level," has been added in Figure 5-1. This new factor is a subjective one, but presumably, if given a quantitative representation, it would help in explaining some of the variation in the variables that are used in our quality index. Types of empirical information that could, perhaps, be used to represent aspiration level are the per cent of families in a community with incomes over, say, 10,000 dollars, the per cent of adults with more than high school education (or with more than four years of college), the growth rate of the community, the per cent of families with children of school age, and the voting behavior, in recent elections, of the populace when the issue was related to the public school system (such as a school bond election). (See also the Appendix for comments on data refinement.)

To assist us in determining an appropriate form for 5-3, a closer analysis of some of its components will be made in the following sections. First, a conceptual framework for determining teachers' salaries is presented. Next, we analyze in turn the factors that may influence the number of credit-units offered, the number of assignments per teacher and the number of college hours per teaching assignment.

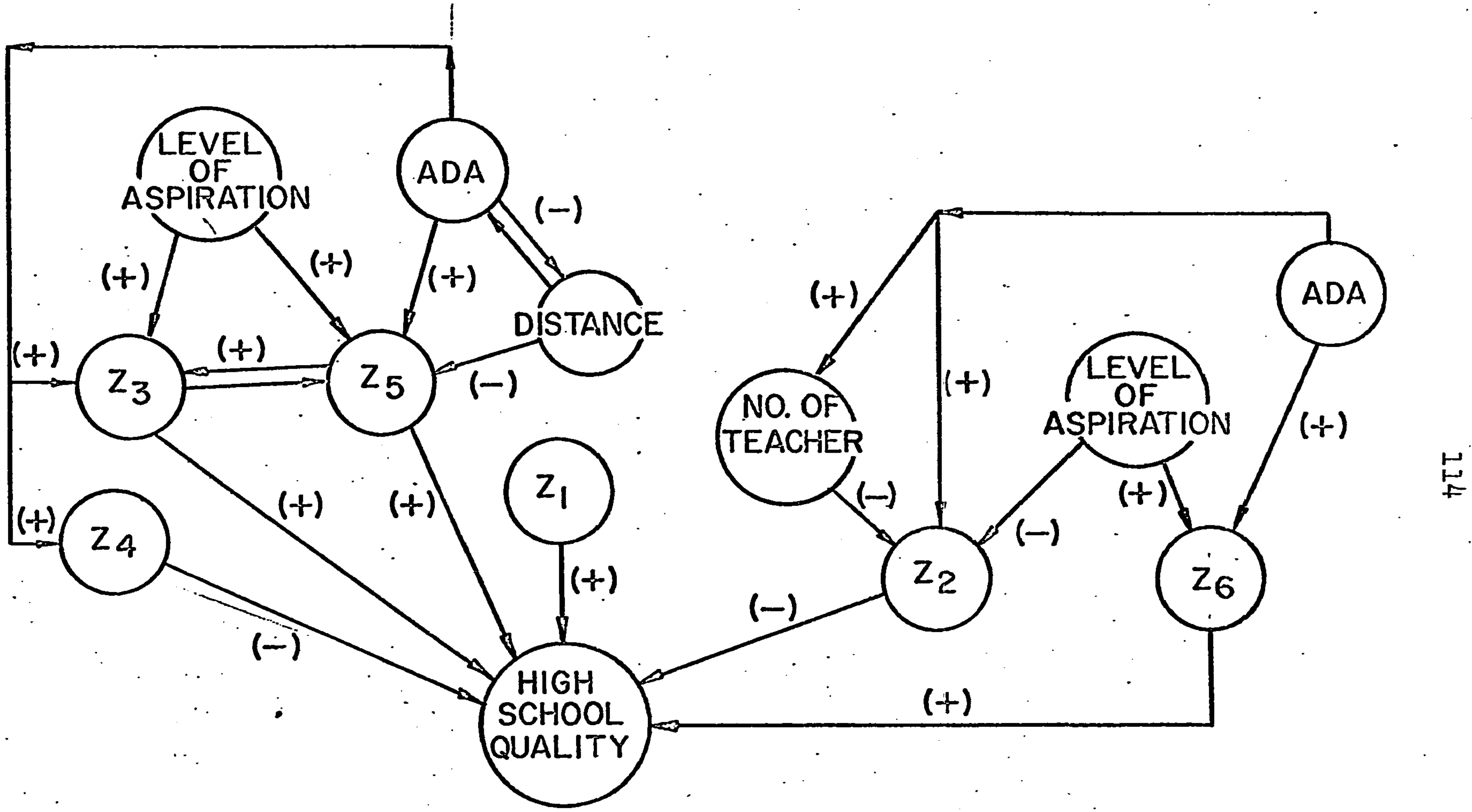

Figure 5-1. Factors affecting high school quality (see text for the definition of Z_i)

In addition, some comments are made concerning class size and other factors (the availability of teaching aids, supervisory personnel, and the condition of the plant). Finally, we discuss possible adjustments in variables and the estimation of weights for the "adjusted" quality index.

Analysis of Teachers' Salaries

To what extent is the market for teachers similar to other labor markets? Intuitively, while we would expect teachers to respond to the best alternative that may exist in their area of residence, and even, sometime, to move to the area in which the most attractive position may be found, it is also quite possible that many non-economic factors (non-pecuniary in nature) serve to distinguish teachers from other (public or private) employees. For example, the fact that a large majority of the teachers in the elementary and high schools are women, often not the sole family supporters, illuminates the point. That is, teachers whose incomes are merely supplements to the earnings of the head of the household may prefer to stay at schools which are near their homes, in a familiar community, rather than travel several miles to locations where better job opportunities may exist. Furthermore, some teachers will travel a certain distance away from their present home towns to schools with which they have traditionally been associated as parents, teachers or pupils, even though better opportunities yet exist in their immediate vicinity. Such behavior can easily be explained in terms of psychological preference, which economists must take into account.

The typical teachers' salary schedule is structured on the basis of the following elements: (1) the so-called "base salary," (2) the number of years of experience the teacher has, and (3) the educational background of the teacher. In the present context, only the median salary is available, while, theoretically, our main interest lies in explaining differences in the base salary (see the Appendix for more detail).

Locational factors in teachers' salaries

Home-to-work commuting fields in the 1960's are usually centered on urban places exceeding 25,000 in population. Suppose that one can drive, on the average, a distance of 50 miles in one hour. Also, assume that few people will drive longer than 60 minutes from their homes to their places of work. Then, according to analyses made by Fox (13, pp. 5-8), due to the way in which highways have been built in the Midwest (Iowa in particular), a commuting field will cover an area of about 5,000 square miles. Such a commuting field is called a functional economic area (FEA). For Iowa, a map defining the 12 FEA's has been prepared by Fox and is reproduced here as Figure 5-2.

Suppose, now, that a person is able to make a choice as

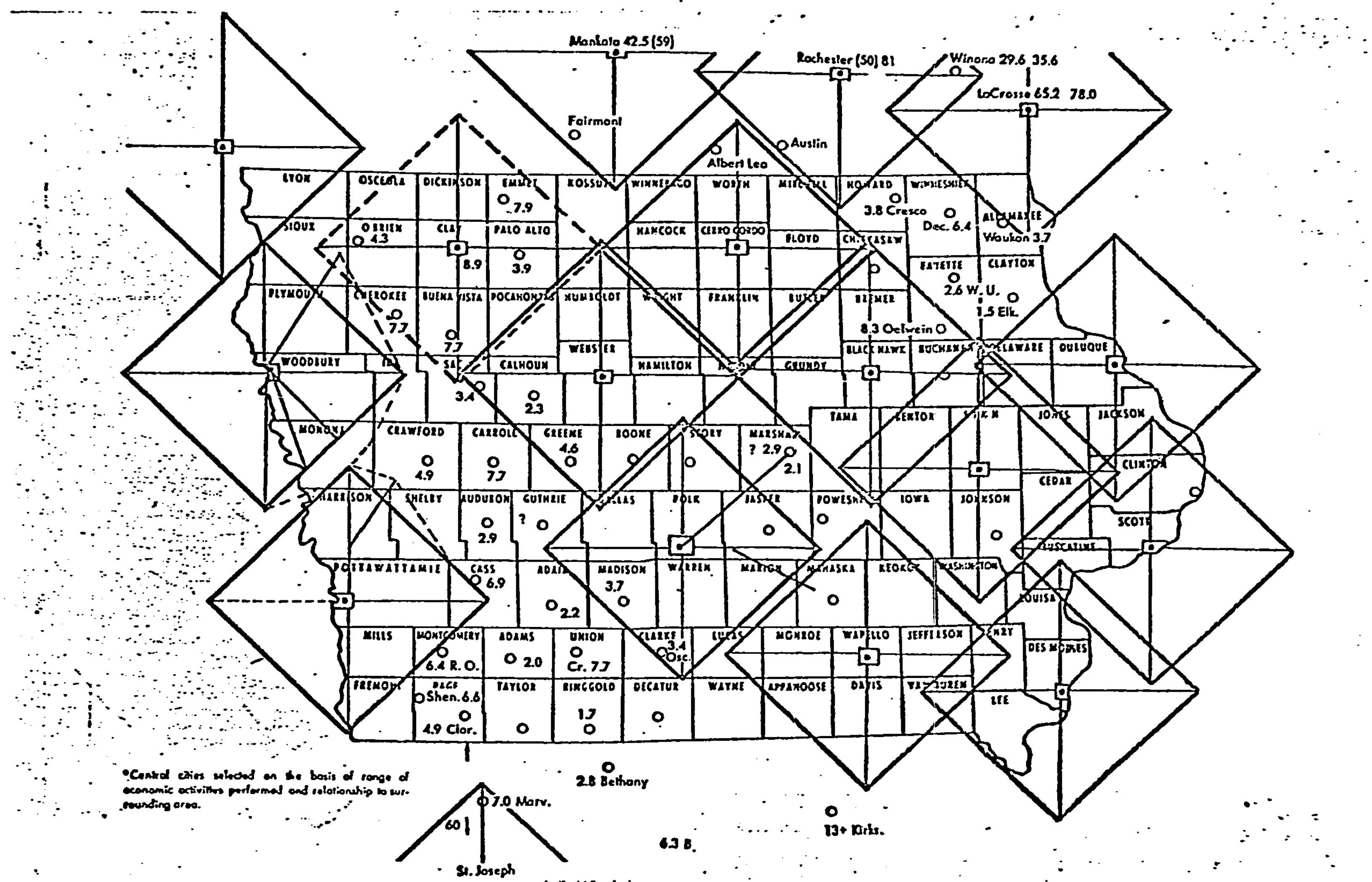

Figure 5-2. Fifty-mile commuting distances from the central business districts of all FEA central cities in or near Iowa. Adapted from Fox (13, Figure 8, p. 35)

to where he should reside within a particular FEA. Suppose, further, that labor markets are perfectly competitive, and that the overriding factor involved in the individual's decision as to the choice of the place to work is dictated by economic (i.e., pecuniary) considerations. In addition, let the cost of commuting per mile be 5 cents. Therefore, if the laborer lives 10 miles away from his workplace, and if he works 50 weeks per year, in each of which there are 5 working days, then the total commuting cost is $5.00 per week, or $250.00 per year (for the daily round trip).

If all other things remain equal, a rational worker will not choose to travel the 10 miles each way each day unless he is compensated an extra amount of $5.00 per week. To sum up, since most of the commuting in a given FEA is from the outlying areas to the central city and back, one may expect the salaries in the central city to be the highest in the particular FEA. And the further a community is from the central city, the less the salary that must be paid to workers of the same type and quality.[1]

A similar hypothesis may be made with respect to teachers' salaries. In general, one would expect the highest salaries

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[1]Farm population in Iowa has been declining for more than 25 years. Many persons have chosen to continue living in small towns or in the open country and to commute fairly long distances to central cities in which job opportunities are expanding.

to be paid to those in the district closest to the center of the FEA. Ceteris paribus, the further the district from the central city, the lower the salary that we expect the teachers to receive. However, other factors than distance from the central city must be taken into account. For one thing, the educational level of teachers in different districts may not be the same. Thus if in one district the average number of college hours per teacher is greater than that in another district, we would expect a priori that the former district will reward its teachers with higher salaries in proportion to their educational level. In addition, different communities have different levels of aspiration regarding the quality of the teaching and supervisory staff. Also, different community philosophies prevail as to how much teachers should be paid. Finally, the wealthier communities can afford to pay higher wages to their teachers. That is, with the same preference function concerning the choice of, say, paying higher wages to teachers versus spending a bit more on other municipal services, a city which is endowed with more resources will no doubt pay higher wages to teachers than would a city which is not as well endowed.

Let us, then, define the following variables:

Z_1 = distance in miles from the high school to the nearest FEA central city

Z_2 = median high school teachers' salary (in dollars) in the nearest FEA central city

Z_3 = median number of school years completed for the population aged 25 years and older

Z_4 = median family income in the community (in dollars)

Z_5 = per cent of families with income over $10,000

Z_6 = college hours per high school teaching assignment

Z_7 = average daily attendance (ADA) in the high school

Z_8 = median high school teachers' salary (in dollars)

Our basic hypothesis is that the determination of Z_8 depends on the level of teachers' salary in the FEA central city (Z_2) and the distance in miles to that city (Z_1), while, at the same time, we must allow for such factors as the average number of college hours per teacher (approximately, Z_6), the community's economic level (Z_4), and, perhaps, the aspiration level of the community (a combination of Z_3, Z_4 and Z_5). Hence the following model is proposed:

$$(5\text{-}4) \qquad Z_8 = f(Z_1, Z_2, \ldots, Z_7)$$

If an additive multiple regression model is the specific form that 5-4 will take, namely,

$$(5\text{-}5) \qquad Z_8 = a + b_1Z_1 + \ldots + b_7Z_7$$

then we would expect, on purely theoretical grounds, to find the following:

$$b_1 < 0$$

$$b_2, b_3, \ldots, b_7 > 0$$

Of course, the factors outlined above may not be additive, so that it may well be that 5-4 should take quite a different form.

Empirical findings

The full set of 375 high schools (districts) cannot be used for testing all of the hypotheses made above. This is so because data for variables Z_3, Z_4 and Z_5 are available only for towns with populations of 2,500 or over. Nevertheless, a number of variants of 5-5 were attempted, disregarding, obviously, the three missing variables. Results of these attempts are summarized in Table 5-1. Also, Table 5-2 gives the simple correlation coefficients for the variables included in Table 5-1.

The results summarized in Table 5-1 suggest that b_1 is significantly negative, while both b_6 and b_7 are (highly) significantly greater than zero. Yet, the empirical tests have failed to confirm the hypothesis that b_2 is positive. As some of the 375 districts used in this test were farther than 50 miles from the central city, it seemed possible that their inclusion may have led to the negative finding. However, after the data had been so screened as to ignore those schools for which Z_1 exceeded 50 miles b_2 was still far from being statistically significant. Information on the socio-economic conditions prevailing in the school districts was not used in the equations reported in Table 5-1. If the full model 5-4

Table 5-1. Factors affecting median teachers' salaries (Based on data for 374 Iowa high schools, 1961-62)[a]

Equation	Intercept	Z_1	Z_2	Z_6	Z_7	R^2	F	Standard error of estimate
I	5,445.92 (380.90)	- 5.12 (1.58)	- 0.0004 (0.0564)			0.027	5.31	603.28
II	5,019.95 (352.22)	- 1.64 (1.50)	0.0206 (0.0517)		0.562 (0.065)	0.187	28.53	552.25
III	4,599.37 (351.48)	- 4.07 (1.42)	- 0.0551 (0.0507)	41.08 (4.22)		0.225	35.92	539.32
IV	4,537.23 (339.88)	- 2.11 (1.42)	- 0.0280 (0.0493)	30.88 (4.52)	0.359 (0.068)	0.278	35.65	521.21

[a]Numbers in parentheses indicate the standard error of the coefficients. In all cases the dependent variable is Z_8.

Table 5-2. Simple correlation coefficients for the variables (and data) of Table 5-1.

	Z_1	Z_2	Z_6	Z_7	Z_8
Z_1	1.0000				
Z_2	-0.1476	1.0000			
Z_6	-0.0921	0.1227	1.0000		
Z_7	-0.2655	-0.0063	0.4308	1.0000	
Z_8	-0.1666	0.0242	0.4549	0.4291	1.0000

could be tested, it appeared that the negative results with respect to b_2 might be reversed.

This argument led us to limit the investigation to a sample of high schools, all of which are located in towns with populations of 2,500 or over. For such towns, information on Z_3, Z_4 and Z_5 can easily be obtained from the Census of Population reports for 1960. And since the data for the other variables were compiled for the years 1961-62, the difference in the time periods in which the two sets of statistics were collected need cause us little concern.

Using this new set of data, relating to only 81 school districts, a number of variants of model 5-4 were tested. Some of the results, in all of which Z_8 is the dependent variable, are summarized in Table 5-3. Once again, a correlation matrix for variables Z_1, Z_2, ..., Z_8 is presented in Table 5-4.

While highest R^2 is given by Equation VII of Table 5-3 and the lowest standard error of estimate by Equation VI, it seems that little is lost (in terms of the reduction in R^2 and the increase in standard error of estimate) when Equation IV is used.

The predicted value of Z_8, $\hat{Z}_8'$, for any specific school district, using Equation IV of Table 5-3, is given by

$$\hat{Z}_8' = 4{,}191.01 - 3.68\, Z_1' + 0.04\, Z_2' + 39.83\, Z_6' + 0.117\, Z_7'$$

where the apostrophes denote a specific value of Z_i. Confidence intervals for $\hat{Z}_8'$ are given by

$$\hat{Z}_8' \pm t_\alpha \sqrt{s^2(\hat{Z}_8') + s^2}$$

where t_α is the tabulated value of t for probability level $1 - \alpha$ (and the appropriate degrees of freedom); s^2 = error mean square; and where

$$s^2(\hat{Z}_8') = s^2 \left(\frac{1}{n} + \sum_{i=1}^{r} c_{ii} z_i'^2 + 2 \sum_{i<j}^{r}\sum c_{ij}\, z_i'\, z_j'\right)$$

Also, the following notation is adopted (2, pp. 202-203):

Table 5-3. Factors affecting median teachers' salaries (Based upon data for 81 Iowa high schools in cities with populations over 2,500)[a].

Equation	Intercept	Z_1	Z_2	Z_3	Z_4	Z_5
I	4,043.17 (392.70)				0.327 (0.070)	
II	4,731.48 (839.86)			- 3.54 (6.50)	0.214 (0.165)	28.10 (31.18)
III	4,202.06 (334.40)					
IV	4,191.01 (645.06)	- 3.68 (2.47)	0.040 (0.085)			
V	5,017.45 (1,108.83)	- 3.91 (3.38)	0.053 (0.099)	- 3.61 (6.67)	0.114 (0.188)	31.67 (31.49)
VI	3,823.54 (806.59)			- 0.05 (5.75)	0.090 (0.151)	15.26 (28.01)
VII	3,803.77 (1,029.95)	- 0.82 (3.08)	0.028 (0.087)	- 0.37 (5.95)	0.073 (0.168)	15.83 (28.53)

[a]Numbers in parentheses indicate standard errors of the coefficients. In all cases the dependent variable is Z_8.

Table 5-3. (Continued)

Equation	Z_6	Z_7	R^2	F	Standard error of estimate
I			0.212	21.28	487.17
II			0.221	7.31	490.43
III	43.15 (9.80)	0.141 (0.041)	0.379	23.87	435.04
IV	39.83 (9.98)	0.117 (0.044)	0.400	12.70	433.21
V			0.239	4.73	491.17
VI	35.36 (10.30)	0.118 (0.043)	0.424	11.06	427.37
VII	34.98 (10.46)	0.115 (0.045)	0.426	7.73	432.60

Table 5-4. Simple correlation coefficients for the variables (and data) of Table 5-3

	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8
z_1	1.0000							
z_2	-0.1635	1.0000						
z_3	-0.2846	0.2598	1.0000					
z_4	-0.6727	0.1708	0.5222	1.0000				
z_5	-0.5637	0.1872	0.5421	0.9018	1.0000			
z_6	-0.3061	0.1419	0.1974	0.3689	0.4111	1.0000		
z_7	-0.4366	0.0598	0.0427	0.3726	0.2875	0.3496	1.0000	
z_8	-0.3944	0.1383	0.2082	0.4607	0.4507	0.5339	0.4750	1.0000

n = number of observations (in our case, $n = 81$)

r = number of independent variables ($r = 4$, in Equation IV)

$z_i' = Z_i' - \bar{Z}_i'$

c_{ij} is the element of the ith row and jth column of the symmetric matrix $C = (Z'Z)^{-1}$ which, in the case of Equation IV is:

$$C = 10^{-7} \begin{bmatrix} 327.50 & & & \\ 1.49 & 0.3900 & & \\ 218.42 & -4.7531 & 5,308.80 & \\ 2.16 & 0.0078 & -5.96 & 0.103 \end{bmatrix}$$

In Tables 5-1 and 5-3 we note that the regression coefficients relating Z_8 to Z_6 are of the same order of magnitude (\$30 to \$40) for the 81 largest high schools as for the entire set of 375. On the other hand, the regression coefficients of Z_8 upon Z_7 are much higher in Table 5-1 than in Table 5-3. This would seem to imply that Z_8 and Z_7 are <u>not</u> linearly related. That is, the greater the ADA, the less the associated increase in Z_8 will be for a given increase in ADA.

To shed some additional light on this phenomenon, a quadratic term for Z_7 has been added to the model giving us the following results (for 375 districts):

$$Z_8 = \underset{(131.72)}{4421.96} - \underset{(1.38)}{1.88Z_1} + \underset{(4.71)}{22.59Z_6} + \underset{(0.183)}{1.137Z_7} - \underset{(0.000050)}{0.000228Z_7^2}$$

(5-6) $(R^2 = 0.315, \ F = 42.51)$

As the quadratic component is significantly negative, the results do tend to support the assertion made above.

A comparison of Tables 5-2 and 5-4 shows a great deal of variation in the values of the various correlation coefficients. In particular, the one between Z_8 and Z_1, call it r_{81}, is -0.17 for the full data set, while it is about -0.39 for the smaller sample (of 81). Further, given the root mean squares for Z_8 and Z_1 for each of the sets, we can calculate the regression coefficient, b_{81} as follows:

$$b_{81} = r_{81} \cdot s_8/s_1$$

where s_8 and s_1 are the respective root mean squares of Z_8 and Z_1.

Our calculation of b_{81} for the two data sets reveals that for the full set b_{81} = - \$5.12 per mile per year, while for the smaller sample, b_{81} = - \$9.64 per mile per year. Assuming that each teacher is required to come to work on 180 days each year (180 "contract days"), and assuming that these are the only days which are taken into account (i.e., excluding special trips for P.T.A. meetings, special seminars and the like), then, on the average, b_{81} = - 2.8 cents per mile per day for the round trip, or - 1.4 cents per mile each way for the full set (375 observations), while for the small sample the figures are 5.4 cents and 2.7 cents respectively.

Let us suppose further that, on the average, one gallon of gasoline costs 34 cents. If, on the average, one could expect to get 15 miles per gallon under normal driving conditions, then the cost of gasoline per mile is approximately 2.2 cents.

The coefficients of Z_1 obtained from both data sets would be consistent with a sophisticated view of commuting costs, including wear and tear, costs of maintenance, and the like. But the standard errors of these coefficients are sufficiently large that we should not claim very much for these results.

In an attempt to gauge the differences in behavior between (1) smaller and larger communites and (2) towns which are nearer the FEA central city and those which are, say, 30 miles away from it, the full data set was divided into four categories as described in Table 5-5. (Set 1 contains all of the districts whose populations exceed 5,000 but which are located not farther than 30 miles from the nearest FEA central city. Set 2 is similar to Set 1, but includes districts that are located within a radius of more than 30 miles from the FEA center. Sets 3 and 4 resemble Sets 1 and 2, respectively, except that they contain districts with populations up to and including 5,000.)

Table 5-6 summarizes the results of two multiple regression models for each of the four sets. The coefficients,

Table 5-5. Averages of teachers' salaries, ADA, distance from FEA central city and other variables (for 374 districts, 1961-62)

Districts' populations	No. of districts				
	Set 1	Set 2	Set 3	Set 4	Total
less than 2,500	0	0	68	97	165
2,500 - 5,000	0	0	42	80	122
5,000 - 10,000	13	36	0	0	49
10,000 - 50,000	19	13	0	0	32
over 50,000	6	0	0	0	6
Total	38	49	110	177	374

Table 5-5. (Continued)

Variables	Means and standard deviations[a]				
	Set 1	Set 2	Set 3	Set 4	All sets
Distance	11.97 (11.25)	48.87 (15.17)	22.06 (14.95)	47.47 (13.29)	36.68 (19.85)
ADA	1,069.21 (1,062.59)	466.20 (244.10)	149.32 (62.24)	156.78 (73.70)	287.79 (450.56)
Median salary	5,920.57 (551.73)	5,608.88 (501.46)	5,100.08 (594.31)	5,111.13 (523.04)	5,255.46 (610.20)
Salary of central city	6,600.01 (466.59)	6,361.48 (654.24)	6,602.01 (615.73)	6,438.50 (563.40)	6,492.81 (558.57)
College hours	35.72 (5.79)	33.18 (4.67)	26.89 (6.10)	26.28 (5.86)	28.31 (6.66)
Assignments per teacher	1.30 (0.34)	1.49 (0.38)	2.51 (0.56)	2.42 (0.54)	2.21 (0.68)
Units offered	49.68 (18.14)	41.76 (6.81)	29.73 (4.58)	29.90 (5.30)	33.41 (10.18)

[a]Numbers in parentheses indicate standard deviations of variables.

Table 5-6. The relationship between median teachers' salaries, distance from FEA central city, and other factors (4 data sets described in Table 5-5)[a]

Equation	Intercept	Z_1	Z_2	Z_6	Z_7	R^2	F	Standard error of estimate
I	2,283.13	- 3.77	0.29	43.60	0.15	0.479	7.60	421.36
Set 1	(1,116.51)	(7.20)	(0.15)	(13.10)	(0.08)			
	4,685.01	- 3.46	0.02	18.70	0.74	0.269	4.05	447.75
Set 2	(903.86	(4.61)	(0.10)	(17.01)	(0.32)			
	4,076.92	- 2.98	0.10	6.18	1.67	0.053	1.49	588.98
Set 3	(735.57)	(3.79)	(0.11)	(9.48)	(0.91)			
	5,387.57	- 6.50	- 0.15	30.83	1.21	0.184	9.72	477.80
Set 4	(452.85)	(2.80)	(0.06)	(6.63)	(0.53)			
II	6,117.46	- 16.44				0.112	4.56	526.90
Set 1	(125.66)	(7.69)						
	5,984.79	- 7.69				0.054	2.69	492.85
Set 2	(239.70)	(4.68)						
	5,161.80	- 2.79				0.005	0.53	595.58
Set 3	(101.52)	(3.81)						
	5,228.58	- 2.47				0.004	0.69	523.49
Set 4	(146.28)	(2.96)						

[a]Numbers in parentheses indicate standard errors of coefficients.

b_{81}, range from - 2.47 dollars to - 16.44 dollars, or from 1.4 to 9.2 cents per roundtrip mile, or 0.7 to 4.6 cents per actual mile traveled. All of these coefficients have the expected sign; however, their standard errors are substantial.

Before we close this section, a number of cautions must be made. In the first place, this analysis is based on median high school teachers' salaries. More appropriately, account should be taken of (1) the base salary, (2) the increments that may be obtained each year, regardless of improvement in the educational background of the teacher, and (3) the increments that a teacher may receive for a greater amount of education. In such a framework, the "distance hypothesis" should be tested with respect to the base salary alone, thus correcting immediately for differences in the average educational background of the teachers.

Second, Z_1 represents the distance in miles with no regard to the quality of the road, the amount of traffic on the road, and to weather conditions in the particular area. In other words, if <u>time</u> rather than <u>miles</u> is the constraint on commuting, one must take into account the variation from area to area in the driving time per mile.

Finally, it must be noted that at best we were able to explain about 48 per cent of the variation in Z_8. This suggests that other variables and data refinements, some of which have already been mentioned, should if possible be included in future models. Also, the coefficients of Z_1 do not appear to be highly (statistically) significant in any of the models. Consequently, any conclusions drawn from this study must be highly tentative at best.

The Rank-Size Rule

Both curriculum breadth and teacher's specialization are generally regarded as components of quality. Holding ADA constant, and assuming that each student has time for a specified number of courses, it is clear that the average class size will decrease proportionately with an increase in the number of courses offered. Hence, if some minimum expected enrollment is required before a new course is offered (say 25 students), there must be, first, a certain increase in ADA before the demand for a new course will be recognized. Furthermore, an increase in ADA will also permit an increase in specialization (i.e., a reduction in the number of courses taught per teacher).

Put another way, we may assume that the students (or their parents) possess implicit indifference maps relating the number of units offered and the average quality per course which may be regarded as a function of the number of assignments per teacher. In other words, given a certain level of enrollment (ADA), a choice can be made between more courses but with less "quality" and fewer courses but with better "quality." Furthermore, the specific level of ADA will define a "budget constraint" in that additional courses can be offered--other things, such as the number of teachers, remaining the same--only at the expense of their quality. In principle, an "expansion path" which is the locus of the equilibrium points for each level of ADA may be derived (18, pp. 12-22). That is, a maximization process is assumed to produce demand curves for both the number of courses and the degree of specialization as functions of the number of pupils in ADA.

Credit-units offered

Assuming that such demands for courses which are not currently offered by a given school are recognized by the school administration, it will be interesting to attempt a reproduction of such demands. To give a specific form to the distribution of latent demands and actual enrollments, we will assume that they follow the rank-size rule. (The distribution may be assumed contingent on teaching of uniform quality in the various courses.) It must be emphasized, however, that we have no clear justification for applying the rank-size rule to either the number of units offered or to the number of assignments per teacher (in the next subsection).

Using the rank-size rule which is described by Brian J. L. Berry (6, pp. 76-77), we may formulate the following hypothesis. Let P_r denote the population of a city of rank r, where all cities are ranked from largest to smallest. (Thus the largest city will have rank $r = 1$, and $P_r = P_1$.) Then we expect to find that

$$P_1 \cdot 1 = P_2 \cdot 2 = \ldots = P_r \cdot r = \text{constant}$$

In other words,

$$P_r = P_1/r^q$$

where q is an exponent which generally approximates unity. So, if $q = 1$,

(5-7) $$P_r = P_1/r$$

Let us redefine P and r to conform to the problem at hand. Specifically, we denote the total <u>enrollment</u> of any particular class (including the number of different sections of the same subject matter) in the high school curriculum by P_r, where r refers to the enrollment <u>rank</u> of that particular class. Thus, if the largest enrollment obtains in the first semester English course, its rank, r, will be 1, and its enrollment will be P_1.

Suppose that the rank-size rule applies to the number of courses offered in the high school. If there are 40 different one-semester courses (in a specific high school) and the enrollment in the smallest (P_{40}) is 25, then

$$P_{40} = P_1/40 = 25$$

Hence it follows that $P_1 = 1{,}000$. Following this rule for all P_r, $r = 1, 2, \ldots, 40$, Table 5-7 obtains.

Let us now assume that the minimum class enrollment is to be 25, that the total enrollment (in all courses) amounts to 10,000 student-courses per year, and that the number of courses, n, is undetermined. Given that the rank-size rule applies, what can we say about n?

First, total enrollment is simply

$$\sum_{r=1}^{n} P_r = \sum_{r=1}^{n} \frac{P_r}{r} = P_1 \sum_{r=1}^{n} \frac{1}{r} = 10{,}000$$

Taking logarithms, we have

$$\text{(5-8)} \qquad \log P_1 + \log \sum_{r=1}^{n} \frac{1}{r} = \log 10{,}000$$

We also know that $P_n = 25$. Utilizing formula 5-7,

$$P_n = P_1/n \quad \text{and} \quad P_1/n = 25$$

Therefore,

$$\text{(5-9)} \qquad \log P_1 = \log 25 + \log n$$

Combining the results of 5-8 and 5-9 it is clear that

$$\text{(5-10)} \qquad \log n + \log \sum_{r=1}^{n} \frac{1}{r} = \log 10{,}000 - \log 25 = 2.60206$$

Table 5-7. Rank-size distribution of one-semester course enrollments ($n = 40$, $P_n = 25$)

Rank (r)	Size (P_r)	Rank (r)	Size (P_r)
1	1,000	21	48
2	500	22	45
3	333	23	44
4	250	24	42
5	200	25	40
6	167	26	38
7	143	27	37
8	125	28	36
9	111	29	34
10	100	30	33
11	91	31	32
12	83	32	31
13	77	33	30
14	71	34	29
15	67	35	29
16	63	36	28
17	59	37	27
18	56	38	26
19	53	39	26
20	50	40	25
		41	24

Ranks	Total enrollment
1-10	2,929
11-20	770
21-30	397
31-40	283
1-40	4,379

The solution of 5-10 will give us one positive value of n. For example, if total enrollment is 10,132 and $P_n = 25$, it can be shown that $n = 80$, the sum of the series $\sum_{r=1}^{n} \frac{1}{r} =$ 5.0, and $P_1 = 2{,}000$. A simple illustration of this case is provided by Table 5-9.

It seems intuitively obvious that total enrollment and the average daily attendance (ADA) should be directly related. For the purposes of this section, we may assume that there are two semesters per year, and that every student takes, on the average, five courses per semester. In other words, if ADA = 100, total enrollment = 10ADA = 1,000 student courses <u>per year</u>.

Suppose, then, that total enrollment = 10ADA and that the rank-size rule applies. What would the effect of an increase in ADA be on n, the number of courses offered? Further, if ADA increases, what per cent of this increase will involve increased enrollment in previously-offered courses and in the newly-offered ones?

If $P_1 = 1{,}000$, and n increases from 40 to 41, it can be shown that

$$\frac{\log \sum_{r=1}^{n} \frac{1}{r}}{\log n} = 0.222$$

On the other hand, if $P_1 = 1{,}000$, and n increases from 80 to

Table 5-8. Rank Size Distributions when n = 80 and P_1 = 1,000

Rank (i)	Size (P_i)	Rank (i)	Size (P_i)
41	24	61	16
42	24	62	16
43	23	63	16
44	23	64	16
45	22	65	15
46	22	66	15
47	21	67	15
48	21	68	15
49	20	69	14
50	20	70	14
51	20	71	14
52	19	72	14
53	19	73	14
54	19	74	14
55	18	75	13
56	18	76	13
57	18	77	13
58	17	78	13
59	17	79	13
60	17	80	12

Ranks	Total enrollment	
1-40	4,379	(see Table 5-7)
41-50	220	
51-60	182	
61-70	152	
71-80	133	
1-80	5,066	

Table 5-9. Rank-size distribution when n = 80 and P_n = 25

Rank	Size	Rank	Size	Rank	Size	Rank	Size
1	2,000	21	96	41	48	61	32
2	1,000	22	90	42	48	62	32
3	666	23	88	43	46	63	32
4	500	24	84	44	46	64	32
5	400	25	80	45	44	65	30
6	334	26	76	46	44	66	30
7	246	27	74	47	42	67	30
8	250	28	72	48	42	68	30
9	222	29	68	49	40	69	28
10	200	30	66	50	40	70	28
11	182	31	64	51	40	71	28
12	166	32	62	52	38	72	28
13	154	33	60	53	38	73	28
14	142	34	58	54	38	74	28
15	134	35	58	55	36	75	26
16	126	36	56	56	36	76	26
17	118	37	54	57	36	77	26
18	112	38	52	58	34	78	26
19	106	39	52	59	34	79	26
20	100	40	50	60	34	80	24
						81	24

Ranks	Total enrollment	Ranks	Total enrollment
1-10	5,858	41-50	440
11-20	1,540	51-60	364
21-30	794	61-70	304
31-40	566	71-80	266
1-40	8,758	41-80	1,374
	1-80	10,132	

81, it can also be shown that (see also Table 5-8)

$$\frac{\log \sum_{r=1}^{n} \frac{1}{r}}{\log n} = 0.1889$$

It appears, therefore, that in the range of $n = 40$ to $n = 80$ $\log \sum_{r=1}^{n} \frac{1}{r}$ increases about 0.222 to 0.189 times as fast as log n.

Suppose, once again, that $P_n = 25$. If $n = 80$, $P_1 = 25n = 2{,}000$. And if n increases to 81, P_1 will increase to 2,025. But if enrollment was 10,000 at $n = 80$, it is now $10{,}125 + 25 = 10{,}150$. That is, when n increases by 1.25 per cent, enrollment in P_1 goes up by 1.25 per cent and similarly for <u>all</u> previously existing courses. Also, 25 students take course $n + 1$ (=81). So, if ADA goes up by 1.50 per cent (since ADA = 1/10 of total enrollment, ADA of 1,000 allows n to be 80; for n to increase to 81, enrollment must increase to 10,150, so ADA must increase to 1,015--an increase of 1.5 per cent), n goes up by 1.25 per cent but only 0.25 per cent of total enrollment (1/6 of the increase) is in the new course. Roughly, then, a one per cent increase in ADA would lead to about 0.83 per cent increase in the number of courses offered and 0.17 per cent increase in the sum of the series $\sum_{r=1}^{n} \frac{1}{r}$.

To sum up, if the rank-size rule applies and if a new course must have a minimum expected enrollment of 25, then $P_{1(n)} = 25\,n$, $P_{1(n+1)} = 25(n+1)$, so that $P_{1(n+1)} = P_{1(n)} + 25$. Further, enrollment in all courses 1, 2, ..., n goes up in the ratio

$$\frac{P_{1(n)} + 25}{P_{1(n)}} = 1 + \frac{25}{P_{1(n)}}$$

so that total enrollment in courses 1 through n increases by $25/P_{1(n)}$ per cent. Finally, the actual number of students in these courses increases by

$$25 \frac{\sum_{r=1}^{n} P_r}{P_1} = 25 \sum_{r=1}^{n} \frac{1}{r} = \frac{1}{n} \sum_{r=1}^{n} P_r$$

so that the change in total enrollment is

$$\frac{1}{n} \sum_{r=1}^{n} P_r + 25$$

An interesting result of this analysis follows. Since P_{n+1} becomes a gradually declining proportion of the change in total enrollment as n increases, if costs per pupil remain constant, the <u>relative</u> cost of adding one new course becomes a smaller and smaller percentage of total resources as ADA increases. Moreover, if--as the analysis of Chapter 4 clearly shows--the costs per pupil decline with increased enrollment (ADA), the relative costs of adding an additional course become smaller yet.

Finally, we must note that for large changes in n different results will be obtained. For instance, if n changes from n = 40 to n = 41, given that P_n = 25, enrollment in courses 1 through 40 goes up by 109.5, and 25 students are added to course 41--so the total change in enrollment is 134.5. In other words, of a one per cent rise in ADA, 0.186 per cent would go into the new course and the rest would go into existing courses. Similarly, it can be shown that when n increases from 80 to 81, 0.165 of each one per cent increment in enrollment would go into the new course. But if n increases from 40 to 80 (where ADA rises, roughly, from 438 to 1,013), about 25 per cent of the increase in enrollment would go into courses 41 through 80, while 75 per cent would be channeled into previously existing courses.

Empirical findings

So far we have concentrated our efforts on the theoretical analysis regarding enrollment, ADA, and the number of units offered. What is, then, the actual relationship between ADA and the number of units offered? If we denote the number of units offered by X_6 (as in Chapter 3), then a number of multiple regression models can be tested with X_6 as the dependent variable and ADA as the sole or most important independent variable. Specifically, the following models have been tested (on the basis of 374 districts):

(5-11) $$X_6 = \underset{(0.37)}{28.19} + \underset{(0.0007)}{0.0181ADA}$$

$$(R^2 = 0.644, \quad F = 675.82)$$

(5-12) $$X_6 = \underset{(2.24)}{32.63} + \underset{(0.0007)}{0.0178ADA} - \underset{(0.0053)}{0.0107X_2}$$

$$(R^2 = 0.648, \quad F = 342.68)$$

where X_2 = total school expenditures per pupil in ADA

(5-13) $$X_6 = \underset{(0.44)}{24.77} + \underset{(0.0016)}{0.03555ADA} - \underset{(0.00000048)}{0.00000549ADA^2}$$

$$R^2 = 0.738, \quad F = 523.79)$$

When ADA = 1,000 (as we assumed above), we can calculate X_6 in Equations 5-11 and 5-13. Also, we can use an equation such as Equation I of Table 4-2 to determine the value of X_2 corresponding to ADA = 1,000. Then the value of X_6 can also be calculated for Equation 5-12 (when ADA = 1,000). Using these numerical substitutions for X_2, X_6 and ADA, we get, first, using Equation 5-11:

(5-14) $$dX_6/dADA = 0.01814$$

$$\frac{dX_6}{dADA} \cdot \frac{ADA}{X_6} = 0.429$$

Similarly, for 5-12 and 5-13 we get, respectively,

(5-15) $$\partial X_6/\partial ADA = 0.0178$$

$$\frac{\partial X_6}{\partial ADA} \cdot \frac{ADA}{X_6} = 0.373$$

$$(5\text{-}16) \qquad \frac{dX_6}{dADA} = 0.0335 - 0.000011ADA = 0.0225$$

$$\frac{dX_6 \cdot ADA}{dADA \;\; X_6} = 0.405$$

In summary, the empirical results, using the Iowa data, show that, for ADA = 1,000, a one per cent increase in ADA is associated with an increase in the number of courses offered of between 0.373 and 0.43 per cent. (We note that in Equation 5-12 total expenditures per pupil were held constant while ADA was allowed to vary. Also, a quadratic component was added to the linear relationship of 5-11 in Equation 5-13, and as a result the outcomes of 5-16 are quite different from those in 5-14 and 5-15.)

Divergence between empirical and hypothetical

The above results do not conform to the hypothetical formulations made above, where we expected the per cent increase in X_6 to be about 0.83 of one per cent for each one per cent increase in ADA. However, it must be realized that there are tremendous pressures on even the smallest high school to offer as many different units as possible. In the first place, the State Department of Public Instruction sets a standard of minimum number (and types) of units that each school should offer. Too great a deviation from the standard for too many years, may cost a school its accreditation by the state concerned. Or a school may not be given accreditation until and unless it satisfies the minimum standard for number of units offered. Second, the community may put political and economic pressures on the high school principal to add more courses, even though the demand for these is far below the 25 limit set above. In particular, since many believe that a high school that does not offer a certain number of courses is necessarily inferior, and that, as a consequence, the graduates from that school may find it difficult to go to college or find good jobs after graduation, they will put great emphasis on the number of courses offered.

With limited resources, small enrollment, and great pressure to add more and new courses, the high school principal will be led, so it seems, to force his teachers to assume a greater burden by teaching, on the average, two to three assignments. Further, classes in many subjects will be quite small, implying high costs per pupil in these courses. Consequently, a deterioration of the <u>quality</u> of each course is almost inevitable. First, many a course will be taught by a teacher who is not competent in that subject matter. Second, to keep total per pupil costs down, some cuts in spending that would not have otherwise been made (for a more limited curriculum) will likely lower the quality of those courses for which demand exceeds 25. In all, while the number of courses offered may not conform to the theoretical requirements of the rank-size rule, it may well be true that the number of courses, divided by a certain index of quality, would. That is to say, increased enrollment may not increase the units offered by much, but courses that were previously offered but which were of poor quality (and hence should not really count as "full" courses) may now be improved.

Assignments per teacher (X_4)

The rank-size rule which we have explored to some extent in the previous section can be of some additional utility in explaining the theoretical variations among schools in the average number of different assignments per teacher.

Suppose, once more, that the rank-size rule applies. Further, let us assume that the student-teacher ratio is constant at 25 to 1 (in fact, the mean student-teacher ratio for the 375 Iowa high schools is 20.81, and the standard deviation is 12.75). Then, utilizing the results already obtained in the previous section, Table 5-10 can be construct-

Table 5-10. A comparison of the theoretical and empirical results concerning the average number of assignments per teacher[a]

I. A hypothetical case

ADA	Total enrollment in courses of ranks 1-20	21-40	41-80	No. of teachers	No. of teachers in courses of ranks 1-20	21-40	41-80
185	1,850	0	0	7.4	7.4	0.0	0.0
438	3,700	679	0	17.5	14.8	2.7	0.0
1,013	7,400	1,358	1,374	40.5	29.6	5.4	5.5

ADA	No. of courses offered	No. of teachers	Courses per teacher (X_4)	ΔX_4	Δ ADA	$\frac{\Delta X_4}{\Delta \text{ADA}}$	$\frac{\% \Delta X_4}{\% \Delta \text{ADA}}$
185	20	7.4	2.7	-	-	-	-
438	40	17.5	2.3	-0.4	253	-0.00154	-0.11
1,013	80	40.5	2.0	-0.3	575	-0.00052	-0.09

[a]For further details consult the text.

Table 5-10. (Continued)

II. Empirical (for the 378 Iowa high schools, 1961-62, Table 3-4 above)

ADA	Δ ADA	X_4	ΔX_4	$\frac{\Delta X_4}{\Delta \text{ADA}}$	$\frac{\% \Delta X_4}{\% \Delta \text{ADA}}$
120	-	2.65	-	-	-
200	80	2.19	-0.46	-0.00575	-0.25
382	182	1.58	-0.61	-0.00335	-0.30
1,178	796	1.20	-0.38	-0.00047	-0.11

ed. It appears, then, that (on theoretical grounds) a one per cent change in enrollment would be expected to produce about one-tenth of one per cent change (in the opposite direction) in the number of courses (assignments) per teacher. On the other hand, if the student-teacher ratio were constant at 20 to 1 (rather than 25 to 1), the expected percentage change in X_4 for a given one per cent change in ADA would be about 0.18 for the ADA range of 185 to 438, and 0.096 for the ADA range of 438 to 1,013.

Next, consider the four sets of data that were used in the construction of Table 3-4 (of Chapter 3). In Table 5-10 the mean value of ADA for each of the sets is recorded, together with the applicable mean value of X_4. From these basic figures, it appears that a one per cent increase in ADA, in the range of ADA = 120 to ADA = 200, would result in about one-fourth of one per cent decrease in X_4. In the range of ADA = 200 to ADA = 382, the percentage change would go up to - 0.30, while for the last range (382 to 1,178) the figure would be - 0.11 per cent.

While some similarities exist between the theoretical and the empirical results, especially for the large-ADA group, we must still reconcile the differences that exist for the smaller-ADA groups (in which the percentage change in X_4 differs by about 0.1 to 0.2 per cent). In fact, it seems that on the basis of the arguments made in the previous section the actual change in X_4 should be greater than that which would be generated by the rank-size rule. For if it is true that smaller schools over-extend themselves inso-far as the number of units offered is concerned, they must compensate for this by having a greater than optimal number of courses per teacher. So when ADA increases, up to a certain point, much of the change in ADA will be associated with the reformulation of the policies regarding X_4, while the change in the number of courses will not be of much import (as these were already above their optimal level). To sum up, for the low ranges of ADA, a one per cent change in ADA is expected to produce quite a large change in X_4--more than we would have expected if schools were to behave precisely according to the rank-size rule. But once the optimal level of X_4 is restored, changes in ADA will have the expected impact on X_4 (and, consequently, also the expected theoretical impact on the number of courses offered).

In addition to the empirical results outlined above, some regression models were tested, some of which are reproduced below (for the 374 Iowa schools):

$$(5\text{-}17) \qquad X_4 = \underset{(0.036)}{2.432} - \underset{(0.00006)}{0.00075\text{ADA}}$$

$$(R^2 = 0.25, F = 124.56)$$

$$(5\text{-}18) \qquad \log X_4 = \underset{(0.029)}{1.130} - \underset{(0.012)}{0.353} \log ADA$$

$$(R^2 = 0.67, F = 745.22)$$

$$(5\text{-}19) \qquad X_4 = \underset{(0.04)}{2.80} - \underset{(0.00015)}{0.00266} ADA + \underset{(0.000000044)}{0.000000604} ADA^2$$

$$(R^2 = 0.50, F = 185.77)$$

$$(5\text{-}20) \qquad X_4 = \underset{(0.21)}{1.45} + \underset{(0.0005)}{0.0023} X_2 - \underset{(0.000067)}{0.000690} ADA$$

$$(\text{where } X_2 = \text{expenditures per pupil})$$

$$(R^2 = 0.29, F = 76.64)$$

These equations are self-explanatory. From Equation 5-18 it is immediately obvious that, on the average, a one per cent change in ADA will produce about 0.35 per cent change in X_4. Further, when we substitute in the other models the mean values for ADA and X_4 (that is, 287.83 and 2.21, respectively), the per cent changes in X_4 for a one per cent change in ADA are 0.097, 0.30 and 0.089 for models 5-17, 5-19 and 5-20 respectively.

Furthermore, both 5-18 and 5-19 indicate the nature of non-linearity involved in the relationship between X_4 and ADA (note that in both cases the "fit" is much better than in 5-17 and 5-20; also, the quadratic term in 5-19 is highly significant, and it adds a very significant amount of explanatory power (R^2) over and above that of the linear term). The nature of the relationship depicted in 5-19 is consistent with the argument made above. That is, for small enrollment (ADA), a change in ADA will exert an important influence on X_4 (so as to achieve an optimal allocation of teachers' talents). As ADA increases, the strength of the (positive) quadratic term will be greater and greater, thus diminishing continuously the influence that a one per cent change in ADA would have on X_4 (since by then, according to the rank-size rule, much of the change in ADA would result in the introduction of new courses, leaving the number of assignments per teacher approximately constant).

Finally, in Equation 5-20 an attempt was made to examine the effects of a change in ADA on X_4 when expenditures per pupil (X_2) are held constant. And while the results of 5-20 indicate that such a model is empirically useful (it certainly is from the theoretical point of view), yet it does not change the results of 5-17 by much, and thus the inclusion of X_2 in models such as 5-18 or 5-19 does not seem to be quantitatively important.

The Quality Model

So far we have analyzed three of the "quality variables" which are to be included in the index, Q, of 5-1 and 5-3. And while we do not claim to exhaust all of the possibilities, it seems that in addition to X_4, X_5 and X_6 (see definition in Chapter 3), one should include (1) college hours per teaching assignment, (2) the class size (student-teacher ratio), and (3) some index of the quality of teaching-aids, supervisory personnel, and the condition of the "plant."

College hours per teaching assignment (X_3)

The basic hypothesis is that a richer educational background will necessarily result in increased school quality, other things being equal. Our empirical investigations show that X_3 is significantly correlated with X_5, X_6, X_8 and X_4. About 38 per cent of the variation in X_3 can be "explained" in terms of these variables in the following equation (for 375 schools):

$$(5\text{-}21) \qquad X_3 = \underset{(3.56)}{21.75} - \underset{(0.59)}{2.83}X_4 + \underset{(0.00054)}{0.00204}X_5 + \underset{(0.0550)}{0.0085}X_6 + \underset{(0.0027)}{0.0078}X_8 - \underset{(0.00000064)}{0.00000158}X_8^2$$

$$(R^2 = 0.379, \; F = 45.04)$$

In simple words, schools that are willing to pay higher salaries can expect to attract better educated teachers (or, conversely, better educated teachers can successfully demand higher wages). Also, schools in which X_3 is greater would tend to have (1) greater enrollment--although the effects of ADA diminish as ADA rises--and (2) fewer different subject matter assignments per teacher. The latter phenomenon can be explained, on the one hand, in the context of the rank-size rule applied above. But one may note, on the other hand, that this may be due to the fact that the more educated teachers will tend to specialize in specific subjects so that they will get the chance to concentrate their efforts on the fields in which their competence is greatest. Put another way, schools that are in the market for teachers with more education are also (on the average) more interested in enabling their teachers to become more specialized and, presumably, more competent.

Class size (X_{10})

As stated earlier (in Chapter 2), the relevance of class size to high school quality is quite a controversial topic among educators. It seems, however, that a tutorial system (with relatively small classes) is the best form of education. At the same time, classes that are somewhat larger can be equally effective if the distribution of students according to levels of intelligence is such that competition will be encouraged. Beyond a certain point, the increase in class size will, we believe, lead to the deterioration of quality. Table 5-11 presents a number of indices which take these considerations into account. Many other possible indices could, of course, be constructed.

Table 5-11. Quality indices for average class size (X_{10})

Class size	Indices (1)	(2)	(3)
1-5	10	15	15
6-10	9	15	15
11-15	8	10	15
16-20	7	7	10
21-25	6	5	5
26-30	5	4	4
31-35	4	3	3
36-40	3	2	2
40 and over	2	1	1

Other factors

The availability of teaching aids, supervisory personnel, secretarial and clerical help, as well as the condition of the plant, would seem to be quality variables of some significance. While it is not the purpose of this study to examine these matters, it would be desirable to assess the value of such things as educational television, programmed studies, speech and language laboratories, and the like, in improving the quality of the high school program. Further, it is beyond doubt that the quality and availability of nonteaching personnel (including clerical and secretarial help) would have some bearing upon the final quality of the high school program. As far as buildings and equipment are concerned, it would seem that better facilities would encourage and stimulate better learning and provide, on the whole, a more cheerful atmosphere. But if new buildings are but a mask for poor quality in other respects, the inclusion of this factor in the quality index may make one believe that a school possesses more quality than it really does.

Adjustments and standardization of variables

As indicated above, some of the variables which compose the quality index 5-1 need some sort of adjustment prior to being included in that index. One of these is X_5, median teachers' salaries, where the adjustment needed may be either for the distance between the district and the nearest FEA center, or for the number of college hours per teaching assignment (X_3), or for a combination of both.[1] We can, then, define a new variable, X_5^*, where D = distance, and where

$$(5\text{-}22) \qquad X_5^* = X_5 - b_d D - b_3 X_3$$

The coefficients b_d and b_3 can be estimated by one of the

[1]See, however, the Appendix for further comments.

regression equations of Table 5-1 or 5-3, or by any other such model. Further, since there exists substantial intercorrelation between X_3 and X_4, X_3 and X_6, and X_4 and X_6, similar corrections for X_3, X_4 and X_6 may be desirable.

In addition, X_{10} may be redefined as in one of the versions of Table 5-11 (or any similar version) to give us an adjusted value, X_{10}^*, of class size.

In any event, it would be desirable to standardize all of the variable components of the index so that all will be defined in term of some "quality units." A common procedure is to subtract the mean and divide by the standard deviation so that

$$(5\text{-}23) \qquad Z_i = \frac{X_i - \overline{X}_i}{s_i}$$

where s_i is the standard deviation of the variable X_i.

In sum, after the necessary adjustments and standardization have been carried through, we are left with a "new" index

$$(5\text{-}24) \qquad Q = f(Z_1, Z_2, \ldots, Z_n)$$

Description versus optimization

Model 5-24 can be used for two purposes. First, if the nature of f is known, one can rank the schools, for which the necessary information is available, from the very best to the very poorest. This process is descriptive in nature. And while it may be quite interesting, as well as of quite great importance in many cases (such as for the purpose of advertising a school to a prospective student, or for allocating funds to schools whether as a reward for excellence or, conversely, as a stimulus for improving those with poor quality), still the more challenging aspect of 5-24 is its usefullness in enabling the policy-maker to choose that set of inputs that will maximize his own objective function. Moreover, the two functions require quite different treatment of 5-24. In the descriptive case, a universal rule for f must be developed, against which all of the schools must be measured. This implies that the definition of quality obtains consensus among all school administrators--or at least the majority will agree with the particular formulation. On the other hand, one can define a different function, f, for each and every district, when optimization is our objective. In a sense, then, the optimization process need not define a unique and unequivocal concept of quality. That may be left to the school board and the individuals who are in the policy-making position.

In fact, the analysis of Chapter 3 has clearly shown that it is extremely difficult to define quality uniquely in terms of either the change in the ITED scores or the 12th grade score--if we regard the variables included in 5-1 as determining quality. This is perfectly reasonable, since if the definition of quality varies from place to place, resulting in different utilizations of the factor inputs, no systematic and clear relationship ought to exist between the inputs and the ITED measure. Naturally, it may be that quality _can_ be defined uniquely, and that the results of Chapter 3 are due, first, to the fact that irregularities exist in the handling and administration of, as well as the preparation for the ITED battery, and second, to improper specification, namely, that we have omitted a number of important variables that would have explained an appreciable portion of the variation in the ITED scores (in statistical terms). Also, our exclusive use of the least squares estimation procedure (using linear and logarithmic versions) may have produced these unfavorable results. Probably a number of factors have contributed to the failure of the ITED as a good quality index, namely, that we do not have a consensus on the meaning of quality among educators, that some important variables are, indeed, missing, and that more refined statistical methods may have produced somewhat better results.

Estimation of weights for 5-24

Returning to the quality index 5-24, the most important question that remains to be solved is the estimation of the "correct" weights. Suppose, first, that we are merely interested in the _rankings_ of schools. In that case, we must assume that one, and only one, set of weights applies to all of the schools under investigation--in other words, we must reach a consensus regarding the proper dimensions that affect quality.

A simple solution is the assignment of equal weights to the factors, as was done by Hirsch (19). Hirsch contends that, first, a doubling of any one weight--for any component--leaves the rankings unchanged. Second, a subjective evaluation by educators familiar with the schools under consideration proved that in no case did the results of their rankings differ appreciably from those which the index produced. Perhaps a better solution yet will involve the opinions of a panel of experts, from whose views and reasoning one could approximate the "correct" weights. But when the optimization problem is at issue, a detailed analysis of the philosophy, opinions and attitudes which prevail in the community should be made. On the basis of such analyses, it may be possible to obtain reasonably reliable estimates of the appropriate weights.

Summary

To sum up, our study has indicated some of the ways by which the policy-maker, whether on the national, state, or local levels of government, can attempt to answer two important questions: (1) On what basis can schools be classified insofar as their (academic) quality is concerned? (2) How shall one proceed to find an optimal allocation of resources in the production of schooling, given a set of constraints as well as a specific educational philosophy that will define a given objective function?

Although we have not gone so far as to illustrate the mechanics of the mathematical processes involved in answering these two questions, ample examples abound in the literature of mathematical programming (particularly linear programming) that illustrate the use of such tools in the solution of problems such as (2) above. In addition, similar techniques have already been used in models of optimal resource allocation in a university department (14, 15, 31), and more work is being done on this subject at the time of writing.

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APPENDIX

Derivation of Formula 2-1

Define the following variables, as in Chapter 2.

c_x = costs of producing a human being

c_o = costs incurred up to the point of birth

k = annual percentage increase in cost

Also, define

$d = c_o \cdot k$

$n = x + 1$

Then, if we have an arithmetic progression with the first element = c_o, the total number of elements = n, and the difference between the ith and (i + 1)st element = d, the sum, S, of the progression is

$$\text{(A-1)} \qquad S = \frac{n}{2} \left[2c_o + (n-1)d \right]$$

Substituting the expressions for n and d we get

$$\text{(A-2)} \qquad S = \frac{x+1}{2} \left[2c_o + xkc_o \right]$$

$$= c_o \left[1 + x + \frac{kx(x+1)}{2} \right]$$

A-2 is identical to Equation 2-1 of Chapter 2.

An Aggregation Problem

The results of our empirical models have shown that, for example, an increase in the median high school teachers' salary is associated with an increase in the general level of high school quality, Y_1 or Y_2. However, some caution must be exercised in drawing such inferences. In particular, it may be inferred, from the above result, that if one school increases the general salary level there is a great likelihood, other things equal, that the quality of that school shall increase too. However, it is not necessarily correct to say that if all Iowa schools raised their salary level (by the same margin) that all of the schools will, indeed, tend to experience increases in the level of school quality. This is, precisely, the aggregation problem.

One must note, in this connection, that the effects of a general salary increase in one state (say, Iowa), are much different from such a raise which takes place across the country. Moreover, in either case the short run and the long run effects will differ. First, suppose that only schools in Iowa raise their salary levels. Then, in the short run, one cannot predict any _movement_ of teachers from one school to another (even if the short run encompasses more than a year) as a result of the change. Also, because it takes a long time to change jobs and enter new occupations, there will be no significantly important move from other professions to teaching. At the same time, higher salaries may serve as a morale boost, and therefore, _ceteris paribus_, it may contribute to better quality. On the other hand, it is quite likely that such effects will be only of minor significance.

Suppose, now, that a long run analysis is made. If one could isolate the effects of a salary change for all Iowa schools independently of other changes in the composition of all the school inputs (and some changes are almost certain to occur), the result should be, it seems, an increase in quality. For one thing, if teaching pays now relatively more than it did before, marginal students may well choose teaching over other subjects. (In Chapter 2 we have presented Wilkinson's study of enrollment and present value changes in Canada with regard to teaching and engineering. These results do support the present argument.) With more qualified teachers available, it is likely that the state as a whole could benefit. Second, since it is only Iowa (in our example) which raises salaries, there is a great likelihood that, if the salary increase is sufficiently large, teachers from other states will compete for teaching positions in Iowa (as has been the case in California, for instance). Furthermore, as some qualified educators may now be working in higher paying jobs in industry, there is some likelihood that a few of them will try to return to education as well. In sum, it is not unlikely that as a result of a salary increase of a sufficient magnitude, some mobility of factors into education will result--which, it appears, will tend to increase the quality of education in Iowa schools.

Consider, next, the case in which not only Iowa but all of the states in the U.S. increase their salary level. Then the short run effects will be the same as outlined above, in each of the states, while the long run effects will be much different. For while there may be (imperfect) factor mobility within the United States, insofar as high school teaching is concerned, this is not the case for international factor movements in this instance. Hence one of the factors which may serve to increase quality for one state cannot be considered of much import in this context. Yet the likelihood that new and old educators may be enticed to choose teaching as their profession cannot be ignored, and this factor may tend to (slightly) raise the quality of education in all of the United States. (The argument is as follows: if more teachers are available than would have otherwise been the case, schools have in general a larger pool from which they can choose, making it unnecessary, as it is today in many communities, to hire people whose qualifications are inadequate. Also the greater amount of competition for vacant positions will likely result in higher standards and better efforts on the part of the educators who are entering the market.) All in all, while one may qualitatively overcome some of the aggregation problems, the quantitative aspects of the problem cannot so be overcome. That is, the value of the coefficient (of X_5 in this case) cannot be taken at face value when effects are examined for a whole state, region or country.

Teachers' Salary Schedules and Their Significance for "Sampling" Fluctuations in Median Salaries

In practice, the measure of median teacher salary for any school (with two or more teachers) will depend on the following variables:

S_B = base salary paid to teachers holding a Bachelor's degree and with no teaching experience

D = 1, for a teacher holding a Master's degree
= 0, otherwise

E_1 = years of experience (ranging from 0 to n_1)

E_2 = years of experience (ranging from n_1 to n_2)

H_1 = 1, if number of semester credits <u>beyond</u> highest degree equals to or exceeds 15
= 0, otherwise

H_2 = 1, if the teacher does not hold a Bachelor's degree
= 0, otherwise

Note that years of experience could be divided into many groups, but here it is assumed that there are only two such groupings. Similarly, the variable H can become much more nearly continuous, but in practice schools recognize only discrete increments in college credits earned by the teachers.

Using the variables defined above, the salary schedule will be based upon the following equation:

$$(A\text{-}3) \qquad S = S_B(1 + dD + e_1E_1 + e_2E_2 + h_1H_1 - h_2H_2)$$

The lower case letters indicate the per cent increase (or decrease) in the salary of the individual teacher, S, with respect to the salary base, S_B, and these are determined for each school by its board of education.

To illustrate the point, we shall examine a specific salary schedule for one Iowa school district for the academic year 1966-67. The name of the district is withheld. Now, the vector (d, e_1, e_2, h_1, h_2) for that district is equal to (0.10, 0.03, 0.035, 0.03, 0.15). Consequently, A-3 becomes

$$(A\text{-}4) \qquad S = S_B\,(1 + 0.10D + 0.03E_1 + 0.035E_2 + 0.03H_1 - 0.15H_2)$$

Also, $n_1 = 6$, $n_2 = 15$, and $S_B = \$5,000.00$. Table A-1 presents the full schedule.

The above model, for the specific schedule, indicates that allowance is given for up to 15 years of experience. Note, however, that salary increments for the first six years of experience are less (per year) than those for the remaining nine years. This may be an attempt to keep the more experienced teachers within the school, although the extra reward is not very large.

The schedule runs from \$4,250 to \$8,125. The median

Table A-1. Salary schedule for teachers in a specific Iowa school district, 1966-67

Step	No B.A.		B.A. only		B.A. + 15 hrs.		M.A.		M.A. + 15 hrs.	
	% of S_B	$	% of S_B	$	% of S_B	$	% of S_B	$	% of S_B	$
0	85.0	4,250	100.0	5,000	103.0	5,150	110.0	5,500	113.0	5,650
1	88.0	4,400	103.0	5,150	106.0	5,300	113.0	5,650	116.0	5,800
2	91.0	4,550	106.0	5,300	109.0	5,450	116.0	5,800	119.0	5,950
3	94.0	4,700	109.0	5,450	112.0	5,600	119.0	5,950	122.0	6,100
4	97.0	4,850	112.0	5,600	115.0	5,750	122.0	6,100	125.0	6,250
5	100.0	5,000	115.0	5,750	118.0	5,900	125.0	6,250	128.0	6,400
6	103.0	5,150	118.0	5,900	121.0	6,050	128.0	6,400	131.0	6,550
7	106.5	5,325	121.5	6,075	124.5	6,225	131.5	6,575	134.5	6,725
8	110.0	5,500	125.0	6,250	128.0	6,400	135.0	6,750	138.0	6,900
9	113.5	5,675	128.5	6,425	131.5	6,575	138.5	6,925	141.5	7,075
10	117.0	5,850	132.0	6,600	135.0	6,750	142.0	7,100	145.0	7,250
11	120.5	6,025	135.5	6,775	138.5	6,925	145.5	7,275	148.5	7,425
12	124.0	6,200	139.0	6,950	142.0	7,100	149.0	7,450	152.0	7,600
13	127.5	6,375	142.5	7,125	145.5	7,275	152.5	7,625	155.5	7,775
14	131.0	6,550	146.0	7,300	149.0	7,450	156.0	7,800	159.0	7,950
15	134.5	6,735	149.5	7,475	152.5	7,625	159.5	7,975	162.5	8,125

salary of the teachers actually employed in the high school could fluctuate substantially from year to year. Many of the high schools represented in our 1961-62 data had only five to ten teachers. Suppose the salaries in a five teacher high school were \$5,500, \$5,950, \$6,400, \$7,100 and \$7,800; the median salary would be \$6,400. If the \$7,800 teacher resigned and was replaced by one receiving \$5,500, the median salary would fall to \$5,950. Turnover is considerable; thus, the median salaries contain a large stochastic element which goes far to explain the rather large standard error of estimate associated with our salary equations in Chapter 5.

Data Refinement

The statistical information upon which the empirical content of this study is based was provided, in the main, by the Iowa State Department of Public Instruction. Undoubtedly, much more information is available within the school districts. Also, high school principals could collect and keep on file additional statistics that may be useful for models of the type described in this dissertation.

Specifically, we believe that a number of refinements in the data could and should be made, so that models of educational policy for high schools could be tested.

1. We would like some information on the academic preparation of the teachers. But college credits alone are

not sufficient. Some indication of the quality of those credits is important. Stated simply, we may rely upon some accreditation agencies that will list colleges of education, for example, as "satisfactory" or "unsatisfactory." Further, the content of the courses for which credits were earned seems to be quite important. For example, we would like a physics teacher to have a strong background in physics, mathematics, and related subjects--not in gymnastics or music. In addition, some evidence of personal ability and scholastic motivation, such as grades in major and minor subjects, in practice (student) teaching, and, perhaps, in "methods" courses, is desirable.

2. The assignments per teacher variable could be refined as well. Ideally, each teacher should be assigned entirely within a single major field such as chemistry or physics or mathematics, etc.--except to the extent that gifted teachers of, say, physics also wanted to teach a mathematics course now and then for variety. Also, athletic coaches should teach nothing but physical education. In essence, then, we would supplement the assignments per teacher variable with this additional information.

3. Median teachers' salary reflects, in addition to the base salary, the distribution of experience in teaching, the size (in terms of ADA) of the district, and the educational background of the teachers. Evidence on experience--inside as well as outside the district--is certainly called

for. Further, if salaries are to reflect teachers' productivity, some information on teaching effectiveness is desirable. This will include such things as enthusiasm for teaching, resourcefulness in adapting new materials, interest in keeping up with new developments in subject matter field, ease and flexibility in relating to people, and basic self-confidence and self-esteem. And although these are quite difficult to measure, a school principal who is interested in the maximization of the "output" of his educational plant should be able to develop rough indexes that will reflect the quality of his teachers and provide some basis for merit increases in salary.

4. The variable that measures curriculum breadth--the number of credit-units offered--should be supplemented by a variable indicating the degree of communication <u>within</u> groups such as (a) physical sciences, (b) biological sciences, (c) social sciences, and (d) English and "humanities." In other words, it seems that more sections of the same course may contribute to the overall measure of school quality, if communication among the teachers of the same group actually take places. Consequently, a more detailed listing of (a) the number of <u>different</u> units offered and (b) the number of sections of each unit is likely to shed some additional light on the formulation of quality models for high schools.

5. A new variable that would indicate the technological design of buildings and equipment with respect to both instruction and ease of communication among related groups of faculty members would be of some interest. In addition, the amount and efficiency of secretarial, clerical, supply room, "visual aid" and other supporting services to be done for the faculty by non-academic personnel may be revealing.

6. A variable that indicates teaching load is clearly missing. Such a variable--one which will measure the *effective* teaching load in terms of hours needed by each teacher for class contact and preparation--will indicate the amount of time left over for professional development.

7. Finally, some information on the socio-economic structure of the school district's population would be important, including statistics on income, employment, net migration, age distribution, and, in particular, the educational attainment of the adult population. Records of voting behavior on school issues might serve as a proxy for some of this information; however, an analysis of the effects of socio-economic structure on such voting behavior would be needed before such a proxy could be used with confidence.